A THEORETICAL ANALYSIS OF UNSTEADY LAMINAR FLOW OF AIR IN TUBES WHEN SUBJECTED TO ELEVATED INLET TEMPERATURES

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Approved:

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LIST OF SYMBOLS

A	Characteristic area of the tube
B	Defined by Equation (II-6)
c_p	Specific heat
Ci	Defined by Equation (II-39)
Co	Defined by Equation (II-39a)
Cr	Defined by Equation (II-39b)
D	Diameter of the tube
f	Friction factor
g	Gravitational constant
G	Weight flow per area
Gr	Grashoff number
h	Heat transfer coefficient
j	Denotes general element of the tube
k	Thermal conductivity
L	Length of the tube
M	Defined by Equation (II-26)
N	Defined by Equation (II-39c)
Nu	Nusselt number
p	Static pressure
Pr	Prandtl number
Q	Total heat transfer
R	Gas constant
Rey	Reynolds number

t	Thickness of the tube; time
T	Static temperature
w	Weight flow
x	Axial co-ordinate
U	Mean velocity of the flow
Z	Defined by Equation (III-4)

Greek Symbols

α	Thermal diffusivity
β	Bulk mean modulus
Δ	Finite increment
ε	Emmissivity of the tube wall
ν	Kinematic viscosity
ρ	Density
τ	Non-dimensional time
σ	Stephan-Boltzman constant
μ	Viscosity

Subscripts

a	Ambient condition
c	Heat content of the fluid
e	Enthalpy of the fluid
E	Exit of the tube
f	Fluid
fc	Forced convection
i	Inside of the tube
I	Inlet of the tube
j	Denotes general station in the tube

nc Natural convection

o Outside of the tube

r Radiation

w Wall

Superscript

* Non-dimensional value

SUMMARY

The study of pressure losses in small diameter tubing has been the subject of many analytical and experimental investigations over the past years. However, sufficient data are lacking concerning flow with high inlet temperatures when the tube is allowed to transfer heat away from itself.

It is the purpose of this paper to investigate flow in open tubes with elevated inlet temperatures and to analyze theoretically the various factors which contribute to the pressure drop inherent in flow through tubes. The contributing factor which this study will analyze in detail will be concerned with the elevated inlet temperature.

It is shown from the momentum equation that the pressure drop across the tube is proportional to the mean product of viscosity and temperature. The relevant transient heat transfer equations are solved by the use of a finite difference analysis and then non-dimensionalized. The results show how the mean product of viscosity and temperature varies with the system geometry, inlet temperature, heat transfer characteristics of the tube and time.

The effect of these factors is investigated and a correlation equation is developed which relates the mean product of viscosity and temperature to these factors. Finally, the accuracy of the correlation equation is presented.

CHAPTER I

INTRODUCTION

Sensitive pressure sensing devices have many applications in aircraft and missile flights. In missiles they are used for controlling components or determining velocity and altitude. These devices generally consist of a length of circular tubing connected in series to the sensing instrument, which has a finite volume reservoir.

The pressure sensing device is subjected to a transient input pressure, depending upon the type of trajectory flown by the missile. Inherent in the sensing device system is a finite pressure drop across the length of circular tubing connecting the device to the atmosphere; in other words, the sensing device does not measure the true pressure at the particular altitude.

The investigation of this pressure drop inherent in the sensing device system has been the subject of several theoretical and experimental analyses (Ref. 1, 2, 3, 4). However, previous to this time, the effect of elevated extreme temperatures had not been investigated thoroughly. Even though References 1 and 3 accounted for elevated temperatures, they only report investigations concerned with the case of insulated tubing connected to the sensing device. When considering the temperatures encountered by missiles re-entering the earth's atmosphere today, it seems that the temperature of the fluid reaching the sensing device would be high enough to damage the device if the connecting tubing were insulated.

It is the purpose of this investigation to study the effect of high inlet temperatures on the pressure drop inherent in missile sensing device systems, when the connecting tubing is free to transfer heat away from itself. However, this investigation will not completely solve the sensing device system problem since it will be restricted to open systems. By restricting this analysis to open systems the problem is simplified considerably but, yet, the heat transfer characteristics of the flow are still applicable to the sensing device system.

CHAPTER II

THEORY

The mathematical solution of transient flow in pressure sensing systems is difficult due to the number of independent variables associated with the problem; i.e., three spatial co-ordinates and time. Since the type of systems of current interest involve the use of circular tubing, the number of independent variables may be reduced to three (radial and axial position and time) by assuming axial symmetry.

Next, the assumption of one-dimensional flow eliminated the radial position co-ordinate. It is now possible to describe the flow in the tube with two independent variables, these being axial position, x , and time, t .

The dependent variables studied here are temperature, mass flow, system geometry and fluid properties. Assumptions will be made concerning these variables as the theory is developed.

The assumptions made above simplify the relevant equations considerably. However, the governing differential equations for the mass flow and heat transfer are still non-linear, due to the viscous and compressibility effects.

The problem of the mass flow in the tube may be further simplified by assuming that the mass flow past a particular station is independent of the axial co-ordinate and is dependent only upon time. This assumption of a quasi-steady flow results in an uncoupling of the effects of the independent variables and allows the solution of the mass flow to be obtained from a non-linear, ordinary differential equation.

The relevant equations for the heat transfer will be dependent on time and axial position. For these equations it is not possible to uncouple the time and axial position co-ordinates, as was done with the mass flow equations. Thus, the relevant transient equations will be non-linear, partial, differential equations, the non-linearity being due to the variation of the fluid properties with temperature. However, in this investigation some relations will be introduced in obtaining a solution to these equations which are quasi-steady in nature.

A - Quasi-Steady Analysis of the Mass Flow in Tube.--Much of the general characteristics of the flow in tubes has been derived in previous investigations (Ref. 1, 2, 3, 4). Those areas common to this study will be summarized briefly.

First, consider the following general open system

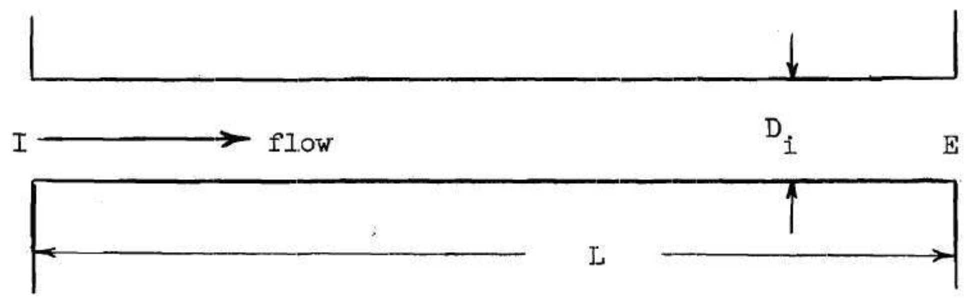

Fig. 1 General Open System

in which there is flow down the tube and where I represents the inlet, E represents the exit, L is the length, and D_i is the inside diameter of the tube.

The momentum equation for one-dimensional steady flow in tubes may be written (Ref. 3),

$$\frac{pdp}{RT} + \frac{G^2 4f}{D_i}\,dx + 2G^2\,\frac{dU}{U} = 0 \qquad \text{(II-1)}$$

where p = static pressure

R = gas constant

T = static temperature

G = weight flow per area = w/A

U = mean velocity of the flow

f = friction factor = $\dfrac{2\tau_w}{\rho U^2}$

w = weight flow

$A = \dfrac{\pi D_i^2}{4}$ = cross-sectional area of tube

As long as the pressure ratio, p_I/p_E , is close to unity, the last term in Equation (II-1) may be neglected. Previous investigations have shown that the pressure ratio is close to unity for the cases of interest here; i.e., laminar flow at atmospheric pressure levels.

Now introduce the approximate relation for fully developed laminar flow

$$4f = \frac{64\,\mu}{G\,D_i}$$

where μ = viscosity

Introducing this relation and integrating between station I and station E yields the following

$$\int_{p_I}^{p_E} 2p\,dp + \int_0^L G^2RT\,\frac{64\,\mu}{G\,D_i}\,\frac{dx}{D_i} = 0 \qquad \text{(II-2)}$$

After integration, cancellation, and re-arrangement, the following relation is obtained

$$p_I^2 - p_E^2 = 64 \frac{GR}{D_i^2} \int_o^L \mu T \, dx \tag{II-3}$$

Now define the following

$$\overline{\mu T} = \frac{1}{L} \int_o^L \mu T \, dx \tag{II-4}$$

so that Equation (II-3) becomes

$$p_I^2 - p_E^2 = 64 \frac{GR}{D_i} \frac{L}{D_i} \overline{\mu T} \tag{II-5}$$

The assumptions inherent in Equation (II-5) are:

1. the flowing medium may be treated as a perfect gas,

   $p = \rho RT$.

2. a quasi-steady state analysis is valid.

3. the flow in the tube is laminar and fully developed.

4. the pressure ratio, p_I/p_E , is always close to unity.

Equation (II-5) may be written in dimensionless form (See Appendix A).

$$\frac{p_I^2 - p_E^2}{p_a^2} = B \frac{L}{D_i} Rey_a \overline{\mu T}^* \tag{II-6}$$

where

$$B = \frac{64 \; R \; \mu_a^2 \; T_a}{D_i^2 \; p_a^2}$$

$Rey_a = G\, D_i/\mu_a$ = Reynolds number of the flow with fluid properties based on the temperature of the ambient conditions surrounding the tube.

$$\overline{\mu T}^* = \overline{\mu T}/\mu_a T_a$$

The subscript a refers to conditions existing in the ambient air surrounding the tube wall such that T_a = constant at all times. For this investigation it will be assumed that the flow of air is discharged into an atmosphere approximating sea level conditions so that $p_a = p_E$.

It should be noted that solving this particular problem does not solve the pressure sensing device problem, since in a missile the flow would exit into a closed sensing volume. However, the solution obtained in this present study will yield information relevant to the heat transfer characteristics of the tube.

Investigating Equation (II-6) it is seen that the pressure drop is a function of B , L/D , Rey_a and $\overline{\mu T}^*$. The term B is essentially independent of the flow in the tube. The effects of L/D and Rey_a have been analyzed adequately in investigations mentioned earlier. However, the effect of $\overline{\mu T}^*$ on the flow has not been developed prior to this study. This investigation will now derive the necessary relations which will allow the determination of $\overline{\mu T}^*$ as a function of the system geometry, heat transfer characteristics of the flow and time.

<u>B - Heat Transfer in the Tube due to Elevated Inlet Temperatures</u>.--The mathematical solution of heat transfer is difficult when the tube is allowed to transfer heat away from itself. The relevant energy equation is a non-linear, partial differential equation of second order. The non-

linearity is due to the variation of the fluid properties with temperature. The solution of this equation will be obtained by the use of a finite difference analysis which follows the procedure given in Reference 5.

To determine the effect of $\overline{\mu T}^*$ on the pressure drop it will be necessary to determine the temperature of the air in the tube as a function of axial position along the tube and to use an adequate representation for viscosity, μ, as a function of temperature. This then will give the variation of μT with axial position along the tube and $\overline{\mu T}$ may be derived from Equation (II-4).

Consider a typical element of tubing of length Δx whose inlet is denoted by j and exit by $j + 1$:

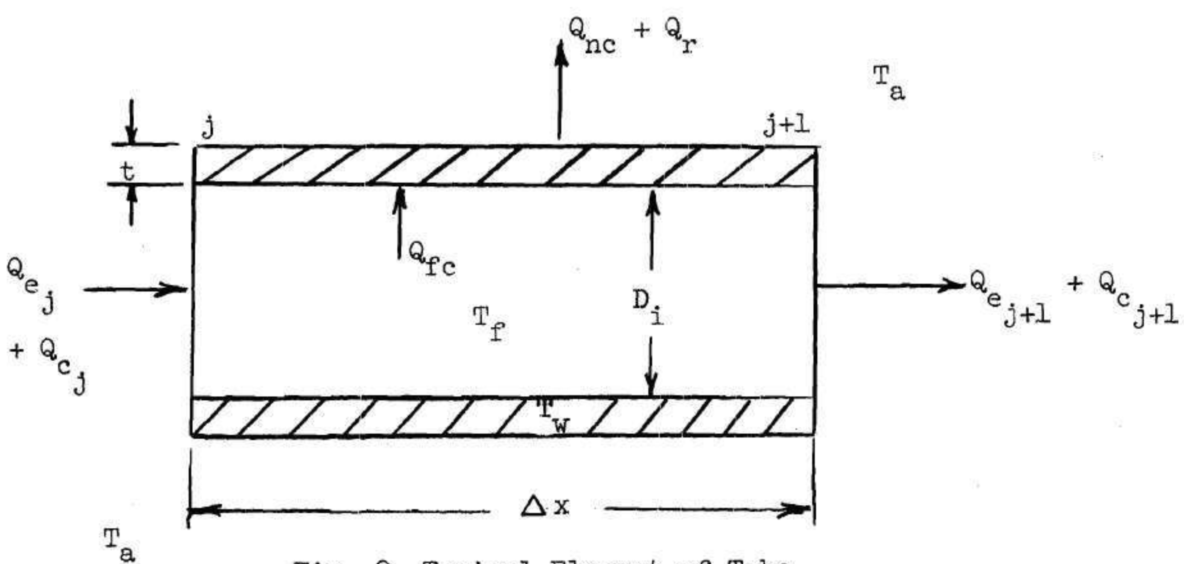

Fig. 2 Typical Element of Tube

where T_a = temperature of ambient air

T_f = temperature of fluid in the tube element

T_w = temperature of the tube wall

Q_e = enthalpy flux of the fluid in the tube element

Q_c = heat content of the fluid in the tube element

Q_{fc} = heat transfer by laminar forced convection

Q_{nc} = heat transfer by laminar natural convection

Q_r = heat transfer by radiation

D_i = inside diameter of the tube

t = tube wall thickness

All heat transfer is in the directions shown in Figure 2.

In discussing a heat balance for the element in Figure 2, the following will be assumed:

1. $w_j = w_{j+1}$; i.e., the mass flow entering station j equals the mass flow leaving station $j + 1$.
2. $c_{p_{f_j}} = c_{p_{f_{j+1}}}$; i.e., the specific heat of the fluid is constant for all fluid elements.
3. properties of the tube wall (ρ_w , c_{p_w}) remain constant.
4. the Prandtl number of the air in the tube remains constant = 0.72.
5. Δx and Δt are sufficiently small that higher differences are negligible.
6. the wall conductivity is large so that T_w is essentially constant through the thickness.

First consider a heat balance for the fluid during a time interval Δt . The general equation may be written as:

The heat flow by forced convection equals the net enthalpy flux plus the rate of change of the heat content of the fluid in the element.

Based on the assumption that small changes in temperature are linear, the difference between the average fluid temperature and the wall temperature during a time interval Δt is approximated as the following relation,

$$\frac{\frac{T_{f_j} + T_{f_{j+1}}}{2} + \frac{T'_{f_j} + T'_{f_{j+1}}}{2}}{2} - \frac{T_{w_j} + T'_{w_j}}{2} \approx \frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}$$

where the prime temperatures exist after Δt time. That is, the temperature difference during time Δt will be taken as its initial value. The effect of this assumption will be increasingly small as Δt is taken smaller. Also, this approximation will not be serious since the prime quantities will be used in the succeeding time interval.

Using these approximations, the heat flow by forced convection is

$$Q_{fc} = h_i A\left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right)\Delta t \qquad \text{(II-8)}$$

where h_i = heat transfer coefficient for forced convection between the fluid and the wall.

$A = \pi D_i \Delta x$

Investigating the net enthalpy flux, $Q_{e_j} - Q_{e_{j+1}}$, the change in temperature of the fluid passing through the element may be approximated as

$$\frac{T_{f_j} + T'_{f_j} - T_{f_{j+1}} - T'_{f_{j+1}}}{2} \cong T_{f_j} - T_{f_{j+1}}$$

Then the enthalpy flux of the fluid becomes

$$Q_{e_j} - Q_{e_{j+1}} = w\, c_{p_f} (T_{f_j} - T_{f_{j+1}}) \Delta t \qquad \text{(II-9)}$$

The change in temperature of the fluid within the element during Δt time is approximated as

$$\frac{T_{f_j} + T_{f_{j+1}} - T'_{f_j} - T'_{f_{j+1}}}{2} \cong T_{f_{j+1}} - T'_{f_{j+1}}$$

in keeping with the previous simplifications. The heat flux due to the rate of change of heat content of the fluid may be written as

$$Q_{c_j} - Q_{c_{j+1}} = \rho_f\, c_{p_f}\, A \Delta x (T_{f_{j+1}} - T'_{f_{j+1}}) \qquad \text{(II-10)}$$

where ρ_f = density of the fluid, and

$A = \frac{\pi D_i^2}{4}$

Combining Equations (II-8), (II-9), and (II-10), the total heat balance equation takes the form

$$h_i \pi D_i \Delta x \left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right) \Delta t = w\, c_{p_f} (T_{f_j} - T_{f_{j+1}}) \Delta t$$

$$+ \rho_f\, c_{p_f} \frac{\pi D_i^2}{4} \Delta x (T_{f_{j+1}} - T'_{f_{j+1}}) \qquad \text{(II-11)}$$

In order to further simplify Equation (II-11) and to uncouple x and t, investigate the order of magnitude of the coefficients of the temperatures in Equation (II-12). First look at the last two terms.

By taking the ratio of the coefficients

$$\frac{w\ c_{p_f}\ \Delta t}{\rho_f\ c_{p_f}\ \frac{\pi D_i^2}{4}\ \Delta x} = \frac{\rho_f\ A\ U\ \Delta t}{\rho_f\ A\ \Delta x} = \frac{U}{\Delta x/\Delta t} \qquad \text{(II-12)}$$

it may be noted that if,

$$\frac{U}{\Delta x/\Delta t} \gg 1 \qquad \text{(II-13)}$$

then the last term may be neglected compared to the second term. This criterion is actually stating that if the time required for the mass of fluid to cross station $j+1$ is small compared to the time interval, Δt , the heat given up as the mass passes station $j+1$ will be small compared to the total heat given up by the fluid element during time Δt . Since the magnitude of Δx and Δt are at the discretion of the investigator, this condition can be assured with any velocity, U , by making Δx suitably small.

Since the magnitude of h_i is not known, the first term may not be neglected; i.e., it must be assumed to be at least as important as the second term.

Therefore, the final equation for the heat balance for the fluid takes the form

$$h_i\ \pi\ D_i\ \Delta x\left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right) = w\ c_{p_f}\ (T_{f_j} - T_{f_{j+1}})$$

or

$$T_{f_{j+1}} = \left[\frac{2\ h_i\ \pi\ D_i\ \Delta x}{2\ w\ c_{p_f} + h_i\ \pi\ D_i\ \Delta x}\right] T_{w_j} + \left[\frac{2\ w\ c_{p_f} - h_i\ \pi\ D_i\ \Delta x}{2\ w\ c_{p_f} + h_i\ \pi\ D_i\ \Delta x}\right] T_{f_j} \qquad \text{(II-14)}$$

Equation (II-14) allows the calculation of the temperature of the fluid leaving a fluid element, if the temperature of the fluid entering and the wall temperature are known along with the coefficients of T_{w_j} and T_{f_j} .

For convergence of Equation (II-14) all the coefficients should be positive. Therefore, the system size should be chosen to satisfy the requirement

$$2\ w\ c_{p_f} \geq h_i\ \pi\ D_i\ \Delta x \qquad \text{(II-15)}$$

or

$$\Delta x \leq \frac{2\ w\ c_{p_f}}{h_i\ \pi\ D_i} \qquad \text{(II-16)}$$

This criterion will be investigated further after the introduction of needed relationships.

Convenient correlation numbers when investigating heat transfer problems are the Nusselt number, Prandtl number, and Reynolds number. These numbers are introduced into Equation (II-16) through their definitions:

$$Nu_i = h_i\ D_i/k_f = \text{local Nusselt number of the fluid} \qquad \text{(II-17)}$$

where k_f = thermal conductivity of the fluid.

$$Pr = c_{p_f}\ \mu_f/k_f = \text{Prandtl number of the fluid} = 0.72 \qquad \text{(II-18)}$$

$Rey = G_i\ D_i / \mu_f = 4\ w\ D_i / \pi\ D_i^{\ 2}\ \mu_f$ = Reynolds number based on the tube inside diameter and the local temperature (II-19)

Introducing Equations (II-17), (II-18), and (II-19) into Equation (II-14) and dividing by T_a to non-dimensionalize the temperature and writing $T^* = T/T_a$, yields the following,

$$T^*_{f_{j+1}} = \left[\frac{4\ Nu_i}{2\ Nu_i + \frac{Rey\ Pr}{\Delta x / D_i}} \right] T^*_{w_j} + \left[\frac{\frac{Rey\ Pr}{\Delta x / D_i} - 2\ Nu_i}{\frac{Rey\ Pr}{\Delta x / D_i} + 2\ Nu_i} \right] T^*_{f_j} \qquad \text{(II-20)}$$

For the Nusselt number, Nu_i , in Equation (II-20), the correlation formula for a quasi-steady flow, as given in Reference 6, will be used,

$$Nu_i = 4.36 + \frac{0.036}{\frac{x/D_i}{Rey\ Pr} + 0.0011} \qquad \text{(II-21)}$$

The constant term, 4.36 , in Equation (II-21) is the Nusselt number for fully developed flow. The second term accounts for development losses associated with a Langhaar (Ref. 8) velocity distribution in the inlet regions of the tube.

Now introducing the approximate relation for the viscosity in terms of the temperature of the fluid,

$$\mu_f / \mu_w \cong (T_f / T_w)^{.675} \text{ (see Fig. 3)} \qquad \text{(II-22)}$$

where for this investigation

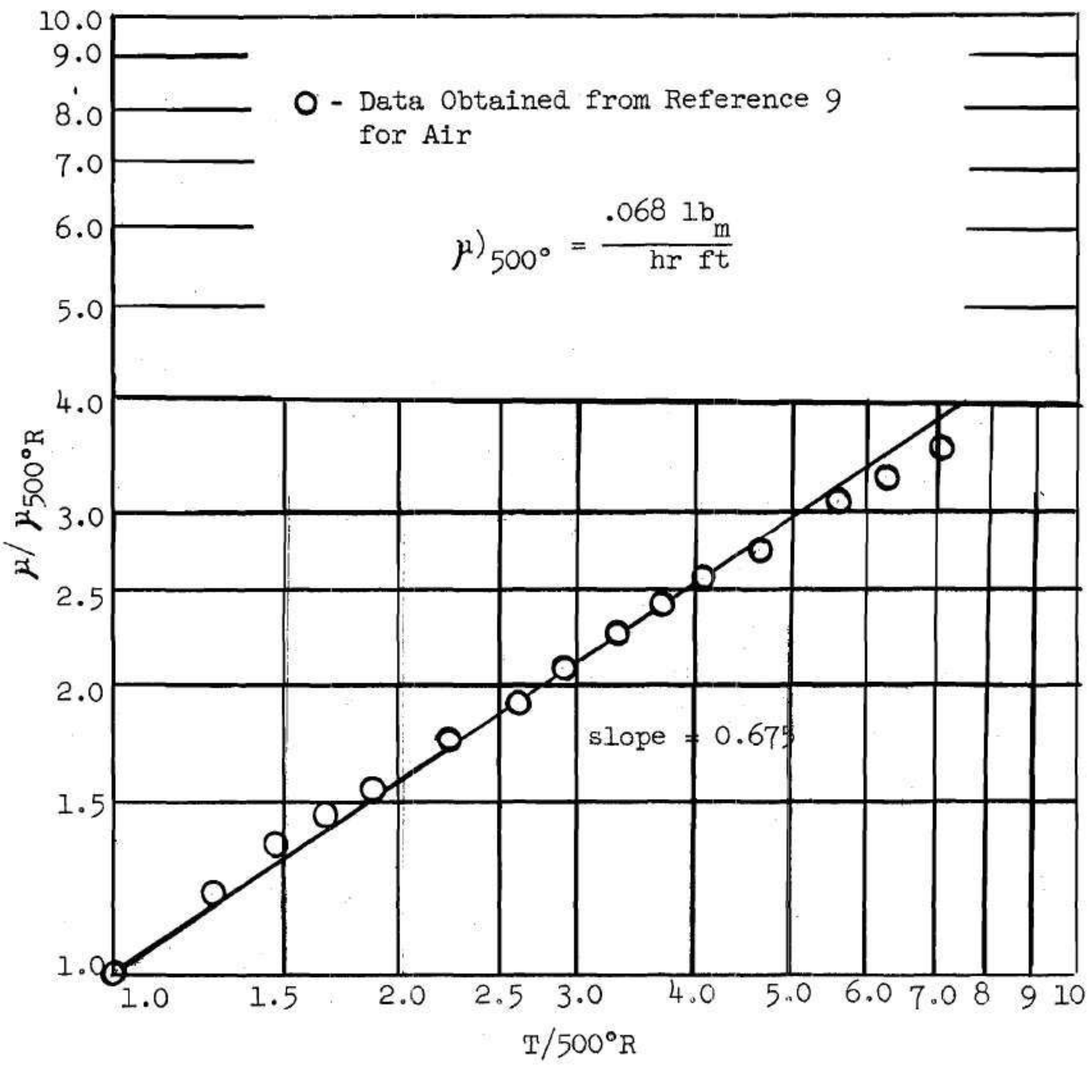

Fig. 3 Variation of Viscosity with Temperature

$$T_f = \frac{1}{2}\left(T_{f_j} + T_{f_{j+1}}\right)$$

as stated earlier.

Since the ratio of Reynolds numbers, Rey/Rey_a , varies as the ratio of viscosity μ_a/μ_f , Rey may be written

$$Rey = Rey_a(T_f/T_a)^{.675} \tag{II-23}$$

Substituting Equation (II-21) and (II-23) into Equation (II-20) yields

$$T^*_{f_{j+1}} = \left[\frac{4\ Nu_i\ M}{1 + 2\ Nu_i\ M}\right] T^*_{w_j} + \left[\frac{1 - 2\ Nu_i\ M}{1 + 2\ Nu_i\ M}\right] T^*_{f_j} \tag{II-24}$$

where

$$Nu_i = 4.36 + \frac{0.36}{(j - .5)M + 0.0011} \tag{II-25}$$

and

$$M = \frac{\Delta x/D_i}{Rey_a\ Pr} \left[.5(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675} \tag{II-26}$$

Equation (II-24) is developed more completely in Appendix B. In Equation (II-26) it appears that $T^*_{f_{j+1}}$ must be known before M can be calculated. However, in the finite difference analysis the quantity

$$\left[.5(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675}$$

will be calculated for the preceding time interval. This error will not be significant as long as the time interval Δt is maintained small.

The criterion for convergence now takes the form,

$$\frac{\Delta x}{D_i} \leq \frac{Rey_a \; Pr}{2 \; Nu_i} \qquad \text{(II-27)}$$

Having now developed the equation relating the temperatures at the inlet and exit of an element, consider the heat balance between the fluid and the wall to obtain the temperatures after a time interval Δt . The following general heat balance may be written for the tube wall:

> The heat transferred to the wall by forced convection from the fluid minus the heat transferred from the wall by natural convection and radiation must equal the heat stored in the wall during a time interval Δt .

The heat transfer to the wall by forced convection from the fluid is, as before,

$$Q_{f_c} = h_i \; A\left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right) \Delta t \qquad \text{(II-8)}$$

The heat transferred away from the wall by natural convection is

$$Q_{nc} = h_o \; A(T_{w_j} - T_a) \Delta t \qquad \text{(II-28)}$$

where h_o = heat transfer coefficient for natural convection from the wall to the surrounding air

$A = \pi D_o \Delta x$

D_o = outside diameter of the tube

and where it is again assumed that the average wall temperature during time Δt is

$$\frac{T_{w_j} + T'_{w_j}}{2} \approx T_{w_j}$$

The heat transfer by radiation is written as

$$Q_r = \epsilon \sigma A(T_{w_j}^{\ 4} - T_a^{\ 4}) \tag{II-29}$$

where ϵ = emmissivity of the tube wall

σ = Stephan-Boltzmann constant

= 0.174×10^{-8} Btu/hr^{o}R4ft^2 (Ref. 7)

$A = \pi D_o \Delta x$

The heat stored in the wall or the increase in internal energy of the wall may be written as

$$Q_w = \rho_w c_{p_w} A \Delta x(T_{w_j} - T'_{w_j}) \tag{II-30}$$

where $A = \frac{\pi}{4}(D_o^{\ 2} - D_i^{\ 2})$

and the prime notation refers to T_{w_j} after time Δt .

Now combining Equations (II-8), (II-28), (II-29), and (II-30) into the heat balance for the wall gives the following,

$$h_i \pi D_i \Delta x\left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right)\Delta t - h_o \pi D_o \Delta x(T_{w_j} - T_a)\Delta t$$

$$- \epsilon \sigma D_o \Delta x(T_{w_j}^{\ 4} - T_a^{\ 4}) = \rho_w c_{p_w} \frac{\pi}{4}(D_o^{\ 2} - D_i^{\ 2})\Delta x(T_{w_j} - T'_{w_j}) \tag{II-31}$$

Solving for T'_{w_j} yields

$$T'_{w_j} = T_{w_j} + \frac{h_i\, D_i\, 4\, \Delta t}{\rho_w\, c_{p_w}(D_o^{\,2} - D_i^{\,2})}\left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right)$$

$$- \frac{h_o\, D_o\, 4\, \Delta t}{\rho_w\, c_{p_w}(D_o^{\,2} - D_i^{\,2})}\,(T_{w_j} - T_a) - \frac{\varepsilon\, \sigma\, D_o\, 4\, \Delta t}{\rho_w\, c_{p_w}(D_o^{\,2} - D_i^{\,2})}\,(T_{w_j}^{\,4} - T_a^{\,4}) \qquad \text{(II-32)}$$

Again introducing the following correlation numbers,

$$Nu_i = \frac{h_i\, D_i}{k_f}$$

$$Nu_o = \frac{h_o\, D_o}{k_f}$$ = Nusselt number for heat transfer from the wall to the surrounding air by natural convection

For the Nusselt number, Nu_o , the correlation formula given on page 205 of Reference 7 will be extended to be made applicable to the range of interest of this investigation

$$Nu_o = 0.95(Gr_o \cdot Pr)^{0.2} \qquad \text{(II-33)}$$

where

$$Gr_o = (g\beta/\nu^2)_o\, (\pi\, \frac{D_o}{2})^3 (T_{w_j} - T_a) \qquad \text{(II-34)}$$

where g = gravitational constant

β = bulk mean modulus of air

ν = kinematic viscosity

and the other terms are defined as earlier.

Now introducing the approximate relation for the variation of $g\beta/\nu^2$ for air with temperature, illustrated in Figure 4,

$$\frac{(g\beta/\nu^2)_o}{(g\beta/\nu^2)_a} = \left[1/2\,(T^*_{w_j} + 1)\right]^{-4.45} \text{ (see Fig. 4)} \qquad \text{(II-35)}$$

Equation (II-34) may be written as (see Appendix C)

$$Gr_o = Gr_a \left[1/2\,(T^*_{w_j} + 1)\right]^{-4.45} \Delta T^* \qquad \text{(II-36)}$$

where Gr_a = Grashoff Number based on the temperature difference $(T_I - T_a)$.

Now introduce the approximate relation for the variation of the thermal conductivity, k, for air with temperature. Data obtained from Reference 7 indicates the following non-dimensional relation, illustrated in Figure 5:

$$k_f/k_w \simeq \left[1/2\,(T^*_{f_j} + T^*_{f_{j+1}})\right]^{.875} k_a/k_w \qquad \text{(II-37)}$$

Using this relation and relations derived in Appendix C, Equation (II-32) becomes

$$T'^*_{w_j} = T^*_{w_j} + \frac{C_i}{N}\left(\frac{T^*_{f_j} + T^*_{f_{j+1}}}{2} - T^*_{w_j}\right)\Delta\tau - \frac{C_o}{N}(T^*_{w_j} - 1)\Delta\tau$$

$$- \frac{Cr}{N}(T^{*\,3}_{w_j} + T^{*\,2}_{w_j} + T^*_{w_j} + 1)(T^*_{w_j} - 1)\Delta\tau \qquad \text{(II-38)}$$

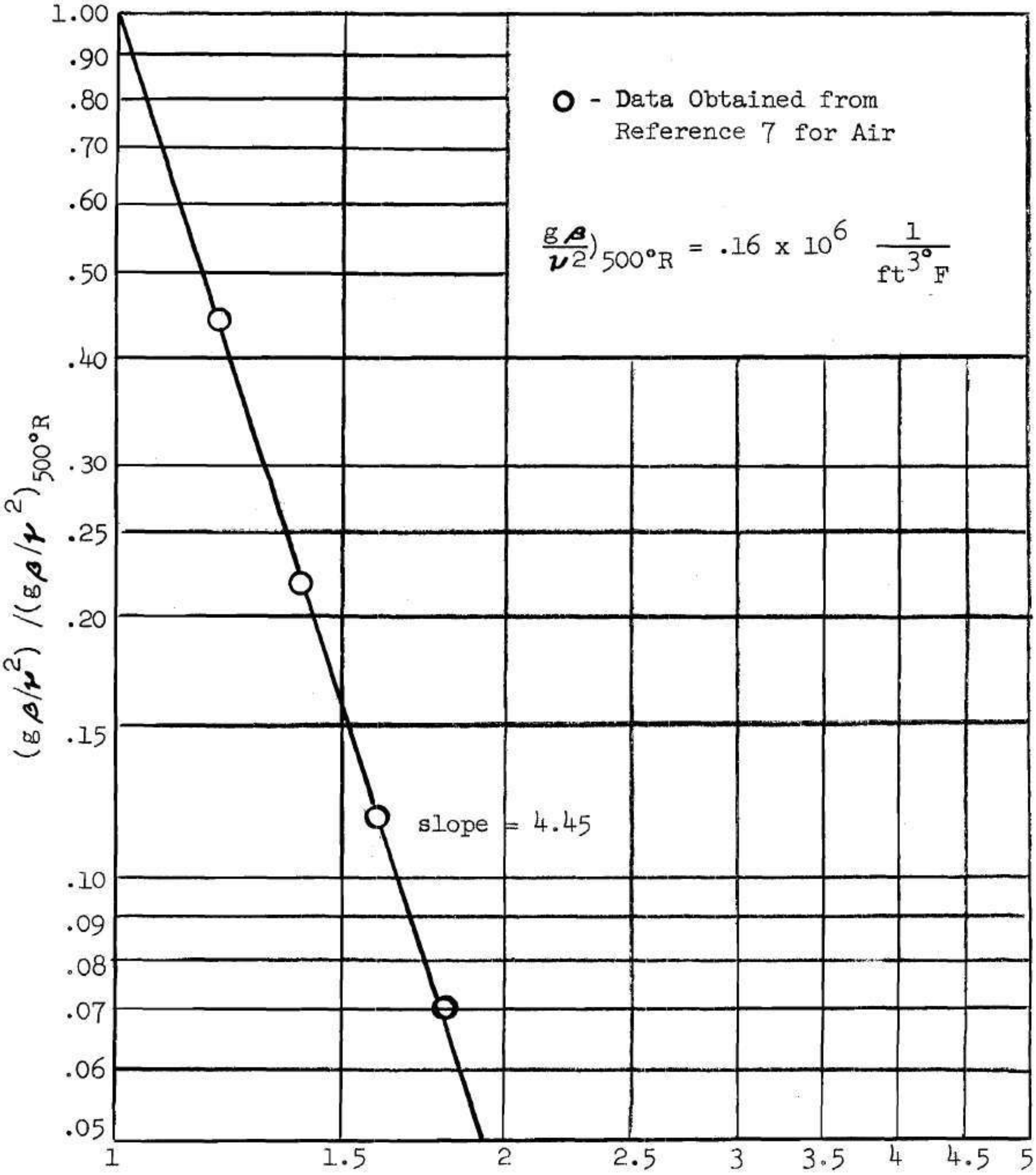

Fig. 4 Variation of $\dfrac{(g\beta/\nu^2)}{(g\beta/\nu^2)_{500°R}}$ with Temperature T/500°R

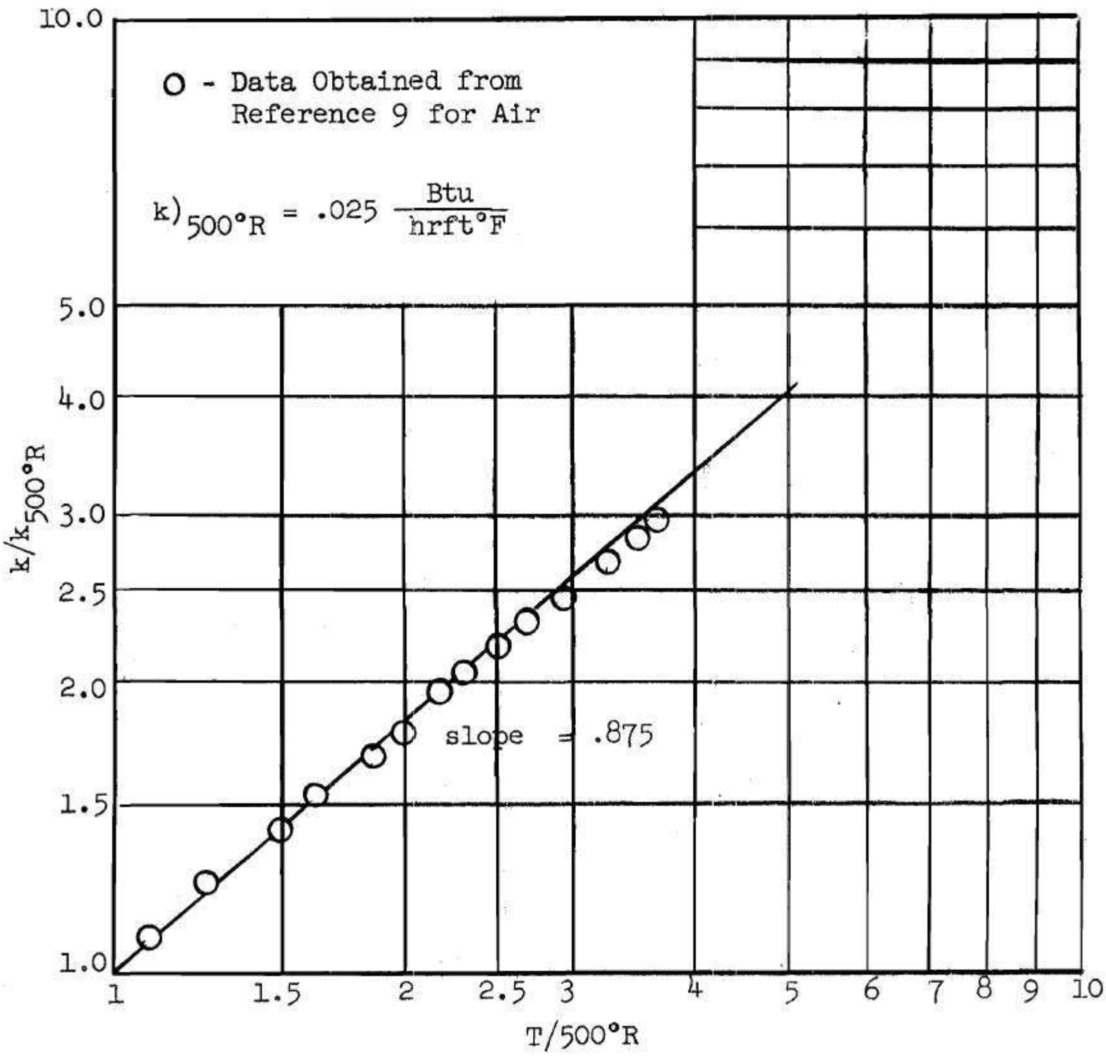

Fig. 5 Variation of $k/k_{500°R}$ with Temperature, T/500°R

where $\Delta \tau$ = non-dimensional time = $\frac{\alpha_w}{D_i^2} \Delta t$, and

α_w = thermal diffusivity of the wall

$$\left.\begin{aligned} Ci &= Nu_i \; k_f/k_w \\ Co &= Nu_o \; k_o/k_w \\ Cr &= \frac{\epsilon \sigma D_o T_w^{\;3}}{k_w} \\ N &= \left(\frac{t}{D_i} + \frac{t^2}{D_i^{\;2}}\right) \end{aligned}\right\} \qquad \text{(II-39)}$$

Equation (II-39) may be re-arranged as follows:

$$T'^*_{w_j} = \frac{Ci}{2N} T^*_{f_j} \Delta\tau + \frac{Ci}{2N} T^*_{f_{j+1}} \Delta\tau + \left[1 + \left(- \frac{Ci}{N} - \frac{Co}{N} - \frac{Cr}{N} T^{*\,3}_{w_j}\right)\Delta\tau\right] T^*_{w_j}$$

$$+ Co\Delta\tau + Cr\,\Delta\tau \qquad \text{(II-40\_}$$

Now for convergence of the finite difference, the coefficients of the temperatures must be positive so that the test for convergence becomes

$$\left(\frac{Ci}{N} + \frac{Co}{N} + \frac{Cr}{N} T^{*3}_{w_j}\right)\Delta\tau \leq 1 \qquad \text{(II-41)}$$

Since the interest is in the minimum value which one must have for convergence, substituting T_I^{*3} for $T_{w_j}^{*3}$ will make the answer conservative since $T^*_{w_i}$ is always less than T^*_I . The criterion becomes,

$$\Delta\tau \leq \frac{N}{(Ci + Co + Cr\, T_I^{*3})} \qquad \text{(II-42)}$$

It now remains to develop the relation for $\overline{\mu T^*}$. As given by

Equation (II-4)

$$\overline{\mu T} = \frac{1}{L}\int_0^L \mu T \, dx \qquad \text{(II-4)}$$

Using the relation derived earlier for the variation of viscosity,

$$\frac{\mu_f}{\mu_a} = \left(\frac{T_f}{T_a}\right)^{0.675}$$

the average value of the product μT between station j and $j+1$ may be approximated by

$$\left[\frac{1}{2}(T_{f_j} + T_{f_{j+1}})\right]^{0.675}\left[\frac{1}{2}(T_{f_j} + T_{f_{j+1}})\right] \cong \left[\frac{1}{2}(T_{f_j} + T_{f_{j+1}})\right]^{1.675}$$

Now approximating the integral in Equation (II-4) with the summation over the interval, the equation for $\overline{\mu T}^*$ becomes

$$\overline{\mu T}^* \cong \frac{1}{L}\sum_{j=1}^{n}\left[\frac{1}{2}(T^*_{f_j} + T^*_{f_{j+1}})\right]^{1.675}\Delta x_j \qquad \text{(II-43)}$$

Thus, the pressure drop across the tube may be calculated from Equation (II-6) for a given tube and inlet temperature.

<u>C - Discussion of Analysis</u>.--As stated earlier, a finite difference analysis has been used to solve for the temperature distribution in the tube. The equations derived in section B for the finite difference analysis are for any general element of the tube.

The tube is divided into increments of Δx length subject to the criterion in Equation (II-27).

$$\Delta x/D_i \leq \frac{Rey\ Pr}{2Nu_i} \tag{II-27}$$

The time increment $\Delta \tau$ is taken according to the criterion in Equation (II-42).

$$\Delta \tau \leq \frac{N}{(Ci + Co + Cr\ T_I^{\ 3})} \tag{II-42}$$

In addition, Δx and Δt must also be of sufficient value to satisfy the criterion given in Equation (II-13).

$$\frac{U}{\Delta x/\Delta t} \gg 1 \tag{II-13}$$

Equations (II-24) and (II-38) may be used in two possible ways:

1. The temperatures, $T^*_{f_j}$ and $T^*_{w_j}$, may be computed for the entire tube at a given time τ_1 and then applied to the entire tube at time $\tau_2 = \tau_1 + \Delta \tau$ and thus continued for the time period desired.
2. The temperatures, $T^*_{f_j}$ and $T^*_{w_j}$, may be computed for the entire time period for the first Δx element of the tube and then applied to succeeding elements of the tube for the entire time period.

This study used the first of these methods for computation of the temperature distribution.

D - Boundary and Initial Conditions.--This investigation will consider the following boundary and initial conditions:

Boundary Conditions: T_I = constant at all times.

T_a = constant at all times.

Initial Conditions: at $\tau = 0$, $T^*_{f_j} = 1.0$ and $T^*_{w_j} = 1.0$

for all elements of the tube.

That is, the tube is initially at temperature T_a and then a constant high inlet temperature is applied instantaneously.

E - Parameters to be Investigated.--This study will investigate the effects of the following seven parameters on $\overline{\mu T^*}$. The numerical values of these parameters studied in this thesis are listed in Table 1.

Table 1

Parameters Investigated

Parameter	Values				
T^*_I	1.2	1.5	2.0	3.0	6.0
Rey_a	100	1000			
Gr_a	1.0	50.0	1000.0		
Cr	0.0	0.10×10^{-4}	0.15×10^{-4}	0.20×10^{-4}	
L/D_i	200	400	600	800	1000
t/D_i	.05	.1	.15	.20	

The seventh parameter is dimensionless time, τ , which was varied from 0 to 2000. It was not necessary to continue the time interval since the values of $\overline{\mu T^*}$ were near steady state at $\tau = 2000$. For a copper tube of 0.25 inches inside diameter, this corresponds to approximately 10 minutes of real flow time in the tube.

After choosing the values of T^*_I , L/D_i , and t/D_i which were of interest and which were considered to be typical values for this type of investigation, the values of Gr_a , Rey_a , and Cr were chosen to be typical values for flow in these tubes when immersed in essentially sea level air.

It should be noted here that Rey_a is not the true local Reynolds number of the flow in the tube. The true Reynolds number of the flow may be calculated from Equation (B-1) in Appendix B.

Again, as was the case with Rey_a , Gr_a is not the actual local Grashoff number for natural convection from the tubes since it is based on T_a = constant . The actual local Grashoff number, Gr_o , for the flow may be calculated from Equation (C-7) of Appendix C once the temperature distribution is known.

It is hoped that these parameters may be correlated into a single equation. If this can be done successfully, it will be possible to calculate $\overline{\mu T}^*$, and thus the pressure drop, for a given tube geometry and flow characteristics and time interval, for the case of constant inlet temperature and constant mass flow studied here.

CHAPTER III

RESULTS

As stated in Chapter II, the object of this investigation is to now derive a correlation formula relating the parameters L/D_i , t/D_i , Rey_a , Gr_a , Cr , and γ to $\overline{\mu T}^*$. To obtain sufficient data to accomplish this, the finite difference equations were set up on a digital computer and were investigated for the values of the parameters listed in Table 1.

First a note concerning the radiation parameter Cr . Computer runs were made for the value of $Cr = 0$ to determine the order of magnitude of the radiation effect on the flow. It was observed that the values of $\overline{\mu T}^*$ were as much as 50 per cent higher when the radiation term was neglected. Thus, to get realistic results, the radiation effect had to be included in all the following runs.

Before investigating the possibility of developing a correlation formula, consider the finite difference criteria cited in Chapter II.

<u>A - Finite Difference Criteria</u>.--With these values of the parameters chosen, it was next necessary to determine the finite increments, Δx and Δt . As given in Equation (II-28)

$$\Delta x \leq \frac{Rey_a \ Pr}{2 \ Nu_i}$$

Based on the values of Rey_a equal to 100 and 1000, Nu_i equal to its minimum value of 4.36, and $Pr = 0.72$, the following criteria are obtained:

$$\Delta x/D_i = 82.5 \quad \text{for} \quad Rey_a = 1000$$

$$\Delta x/D_i = 8.25 \quad \text{for} \quad Rey_a = 100$$

In the inlet region of the tube where the temperature of the fluid changes rapidly, $\Delta x/D_i$ was set equal to 10.0 for $Rey_a = 1000$. A short distance down the tube at $x/D_i = 100$, it was set equal to 20.0. Finally, at $x/D_i = 200$, the increment was changed to 50.0. After $x/D_i = 200$ the temperature changes less rapidly and as such does not warrant such small increments. For $Rey_a = 100$, the increments were set equal to 1.0, 2.0, and 5.0 at x/D_i equal to 0, 10, and 20, respectively.

The requirement given in Equation (II-43) that

$$\Delta \tau \leq \frac{N}{Ci + Co + Cr\, T_I^{*3}}$$

is not as simple to analyze. By calculating upper bounds on Ci , Co and Cr for the particular tests, it was found that as long as

$$\Delta \tau \leq 77$$

then the finite difference analysis would converge for all the cases studied. For these particular tests $\Delta \tau$ was chosen to be 50.0. For a copper tube of inside diameter of 0.25 inches and $\alpha_w = 4.35 \frac{ft^2}{hr}$, this would correspond to approximately 15 seconds of flow time in the tube since

$$\Delta \tau = \frac{\alpha_w}{D_i^2} \Delta t$$

Now investigate the criterion given in Equation (II-14) and see if $\Delta\tau = 50.0$ and $\Delta x/D_i = 5$ and 50 for $Rey_a = 100$ and 1000, respectively, satisfy the criterion

$$\frac{U}{\Delta x/\Delta t} = \frac{U\Delta t}{\Delta x} \gg 1$$

Now using the criterion

$$\Delta x/D_i \leq \frac{Rey_a\ Pr}{2\ Nu_i},$$

and using the maximum value that Δx may have; i.e., exactly equal to $D_i Rey_a Pr/2Nu_i$, re-writing U in terms of Reynolds number and $\Delta t = \frac{D_i^2}{\alpha_w}\Delta\tau$, this criterion becomes

$$\frac{U}{\Delta x}\ \Delta t = \frac{Rey_a\ \mu_f}{\rho_f D_i}\ \frac{D_i^2\Delta\tau}{\alpha_w}\ \frac{2Nu_i}{D_i Rey_a Pr} = 2Nu_i\ \Delta\tau\ \frac{\alpha_f}{\alpha_w} \gg 1$$

Using the minimum value of Nu_i ($= 4.36$), $\Delta\tau = 50$, $\alpha_f = 2.0\ \frac{ft^2}{hr}$, which corresponds to $T_f = 600°F$, and $\alpha_w = 4.35\ \frac{ft^2}{hr}$ for a copper tube, this criterion has the value of

$$2(4.36)50(\frac{2.0}{4.35}) \simeq 200 \gg 1$$

Thus, this criterion is also satisfied.

<u>B - Typical Results</u>.--Forty different combinations of the parameters given in Table 1 were analyzed on the digital computer. Since time was not available to analyze all possible combinations of the parameters, only the maximum and minimum values of the parameters were investigated completely.

Some typical results for the fluid and the wall temperature distribution and the variation of $\overline{\mu T}^*$ with τ are shown in Figures 6 through 14.

As was expected, the fluid and wall temperatures increased with τ until some steady state value was reached. This is shown in Figure 6 in which the steady state temperature profile is reached at τ equal approximately 2000.

The variation of Rey_a studied in this investigation had a greater influence on the temperature profiles than any of the other parameters investigated. The effect of Rey_a is shown in Figure 7 for constant values of the other parameters.

To increase Gr_a or Cr is to increase the heat transfer from the tube. Thus, the fluid temperature decreases more rapidly. These effects are shown in Figures 8 and 9 where the fluid temperature is plotted at $\tau = 500$ for the values of Gr_a and Cr given in Table 1. The other parameters are held constant in these plots.

As the thickness ratio, t/D_i, of the tube is increased, its mass is increased so that the tube has a greater heat capacity for removing heat from the fluid. Thus, the fluid temperature decreases more rapidly for increasing t/D_i. This is verified by Figure 10.

The effect of the quantity $Rey_a D_i/L$ on the variation of $\overline{\mu T}^*$ is shown in Figure 11. It is seen that the variation decreases for decreasing $Rey_a D_i/L$. For a constant Rey_a the variation would decrease for increasing L/D_i. This is as would be expected since μT is being averaged over a greater length L as L/D_i increases.

The effect of increasing Gr_a, Cr, and t/D_i is to decrease the

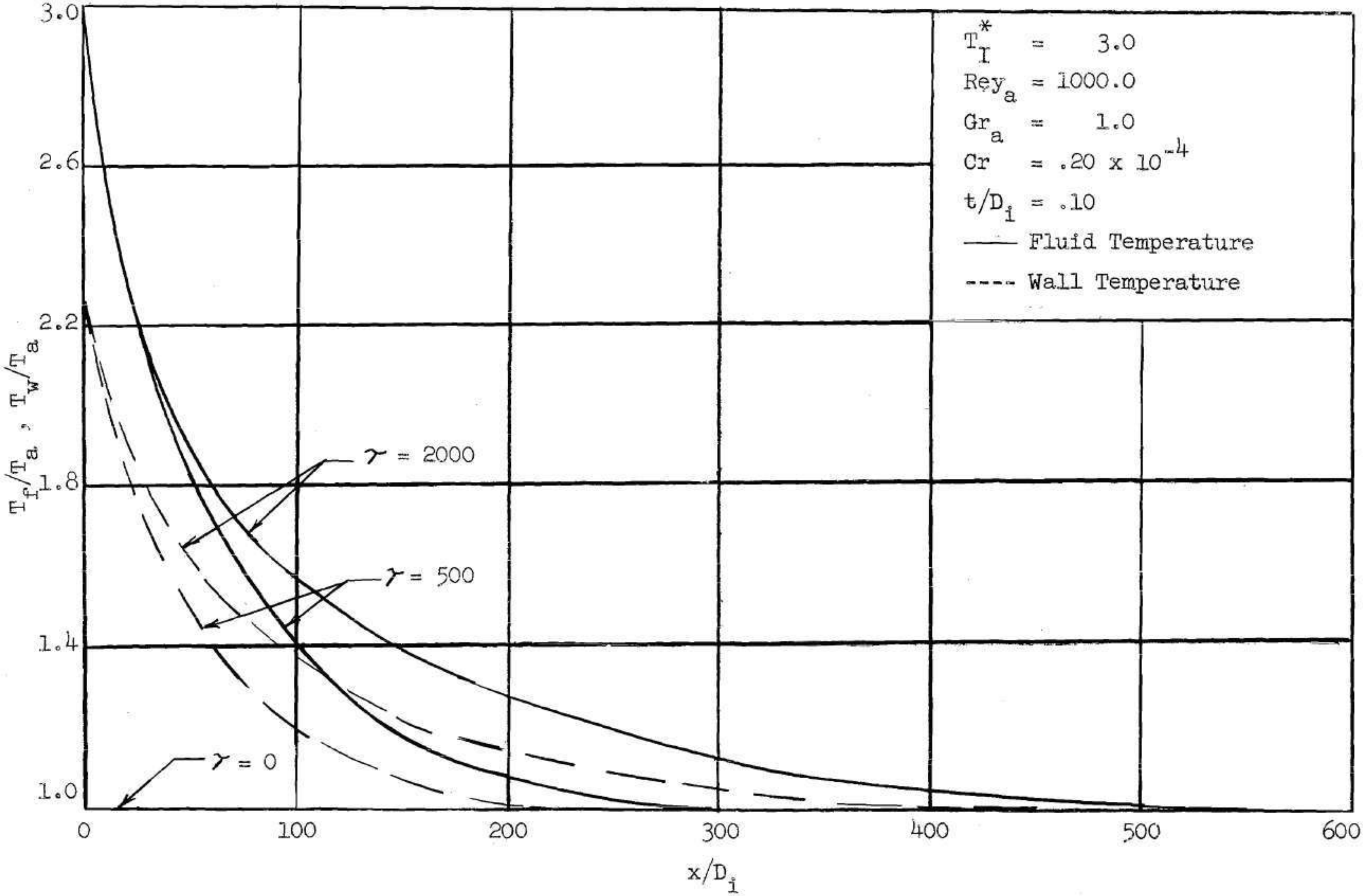

Fig. 6 The Effect of Time, τ, on Fluid and Wall Temperature Distribution

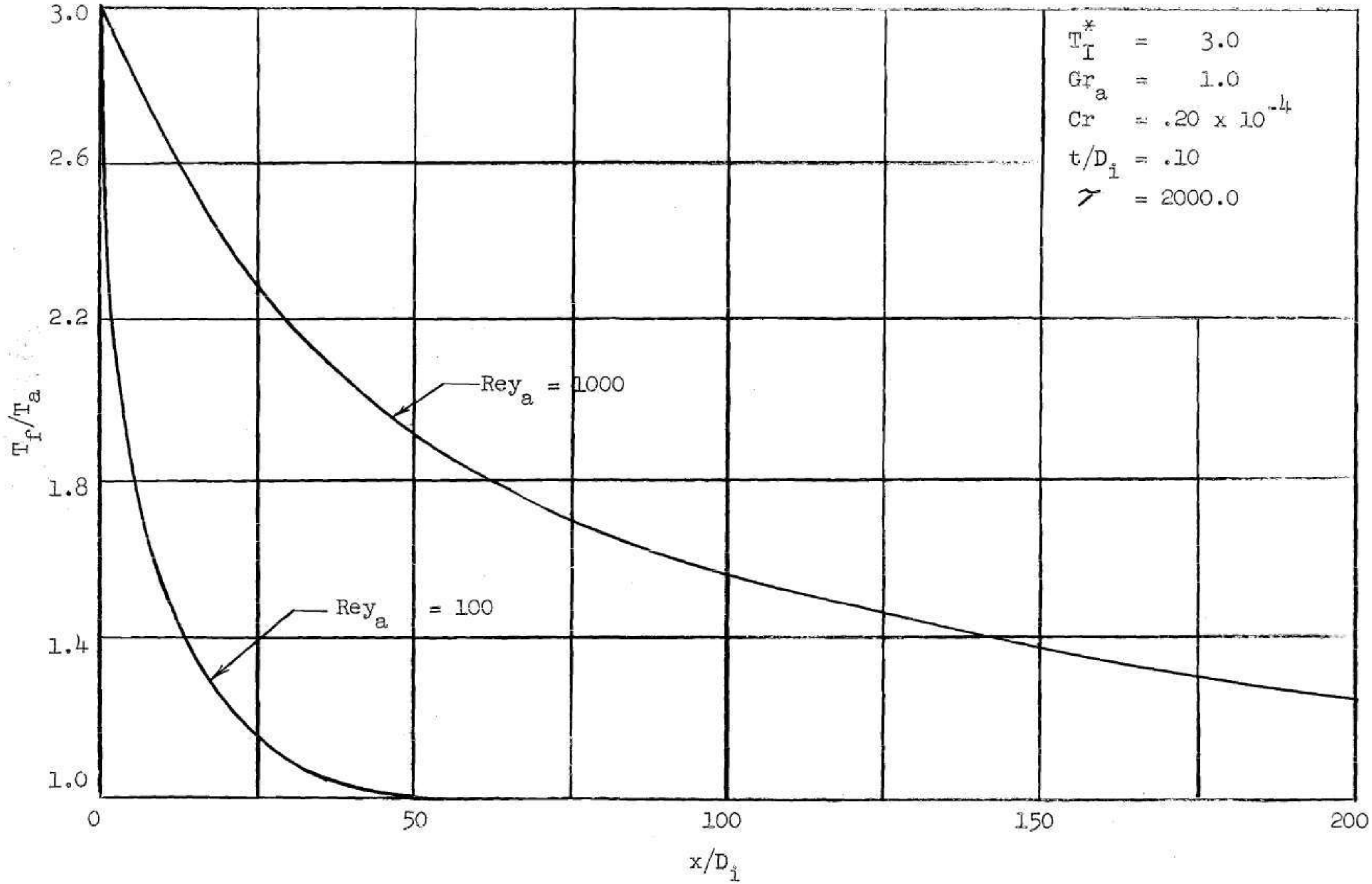

Fig. 7 The Effect of Rey_a on Fluid Temperature Distribution

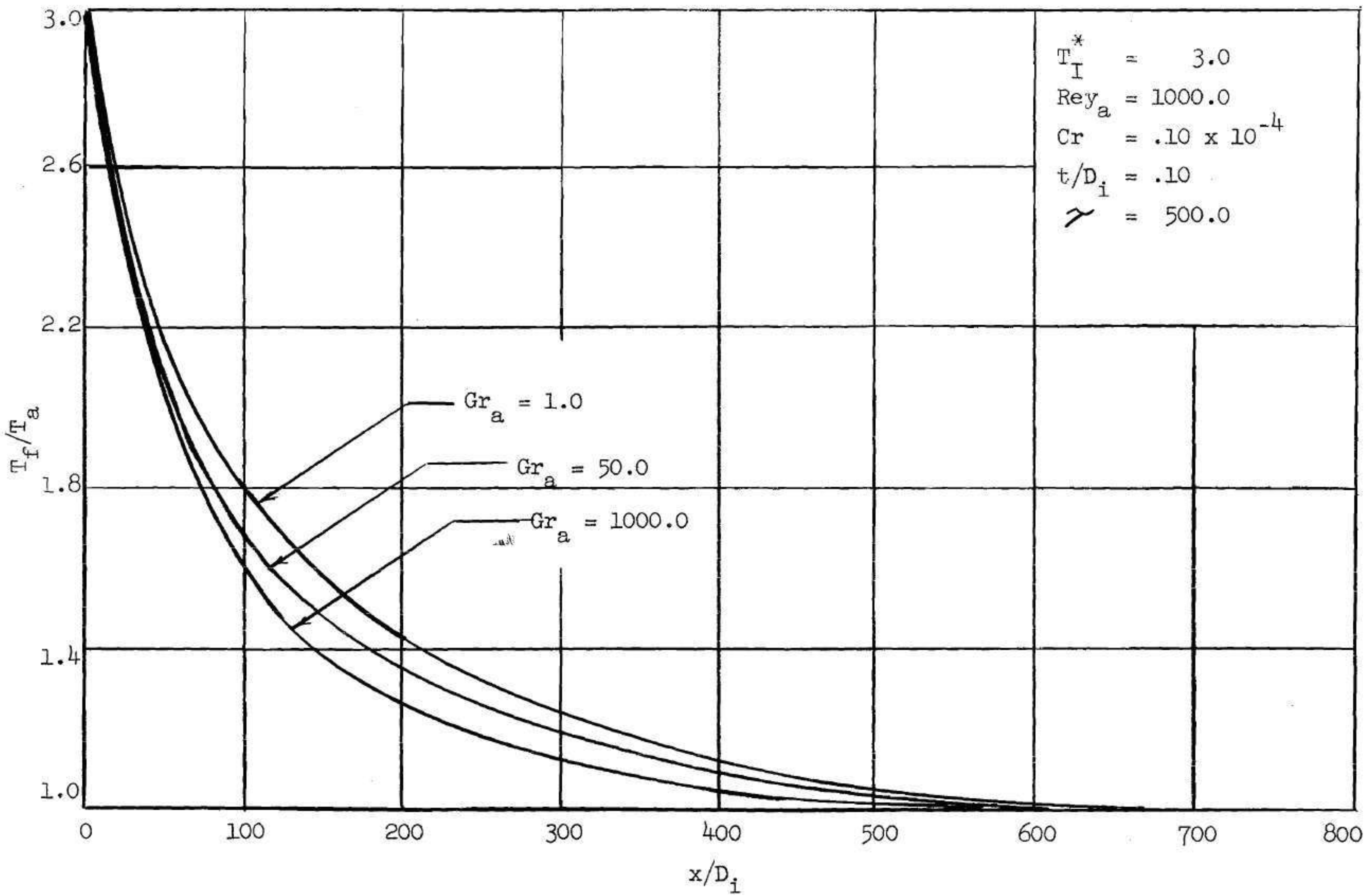

Fig. 8 The Effect of Gr_a on Fluid Temperature Distribution

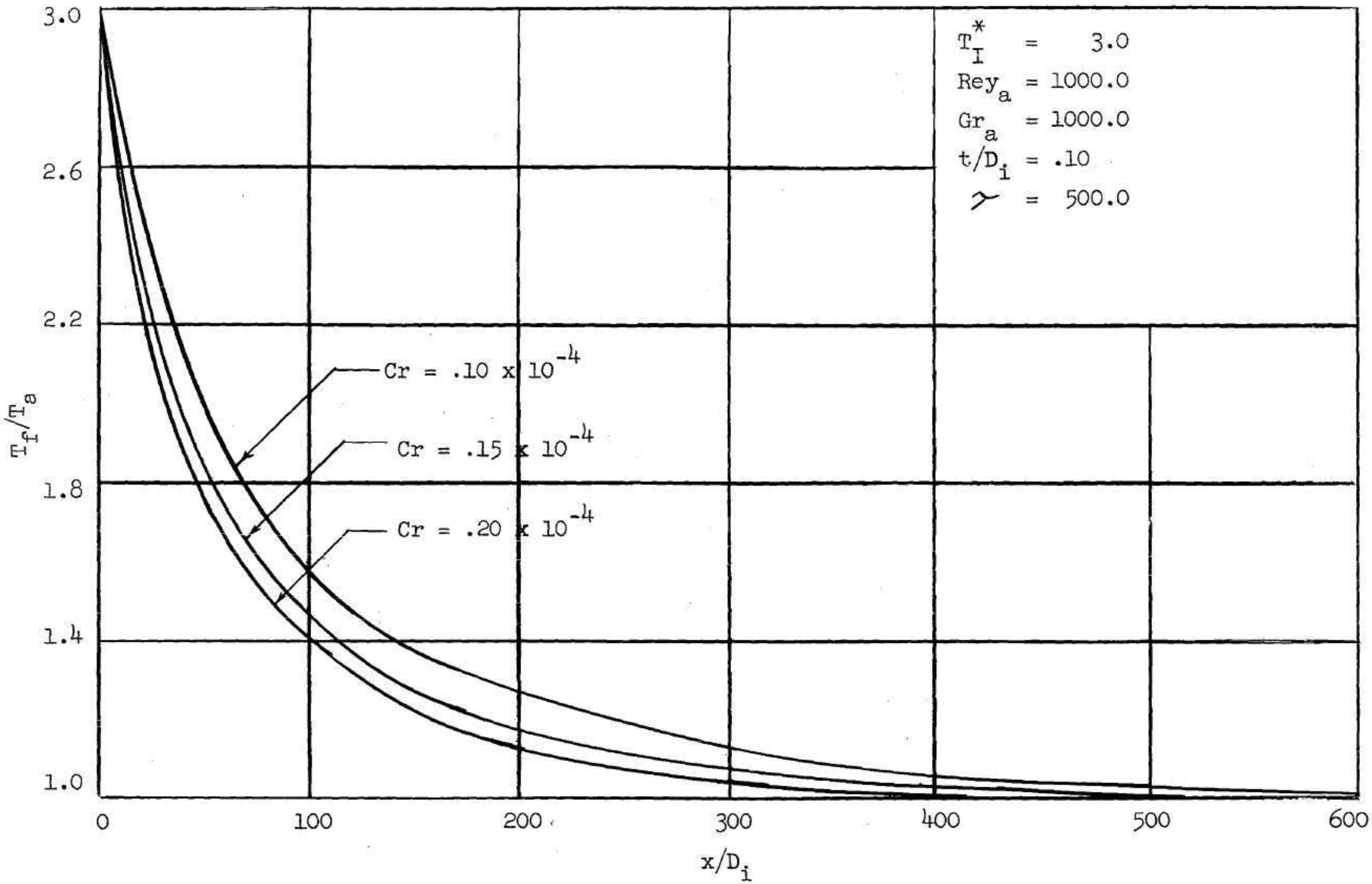

Fig. 9 The Effect of Cr on Fluid Temperature Distribution

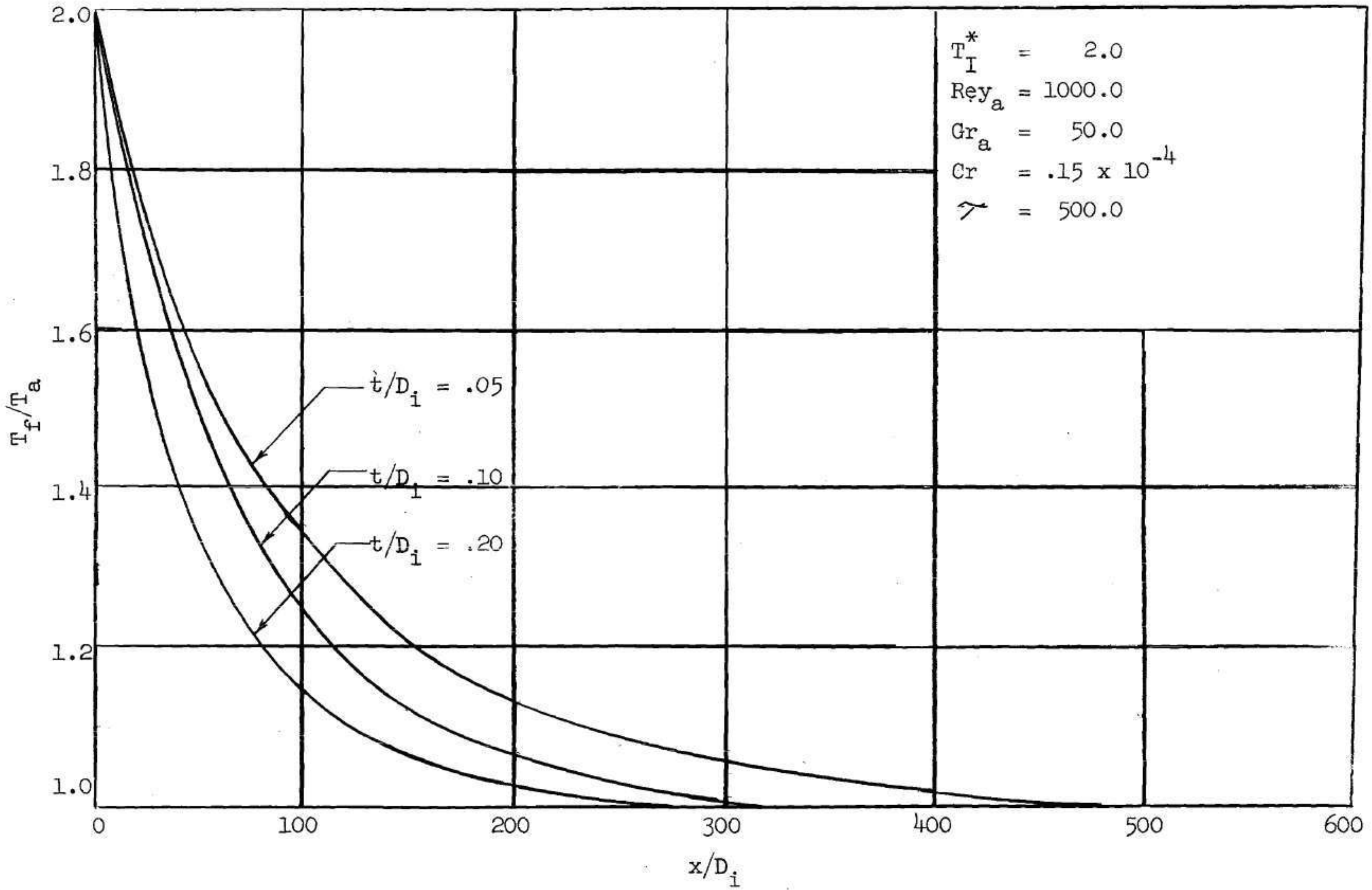

Fig. 10 The Effect of t/D_i on Fluid Temperature Distribution

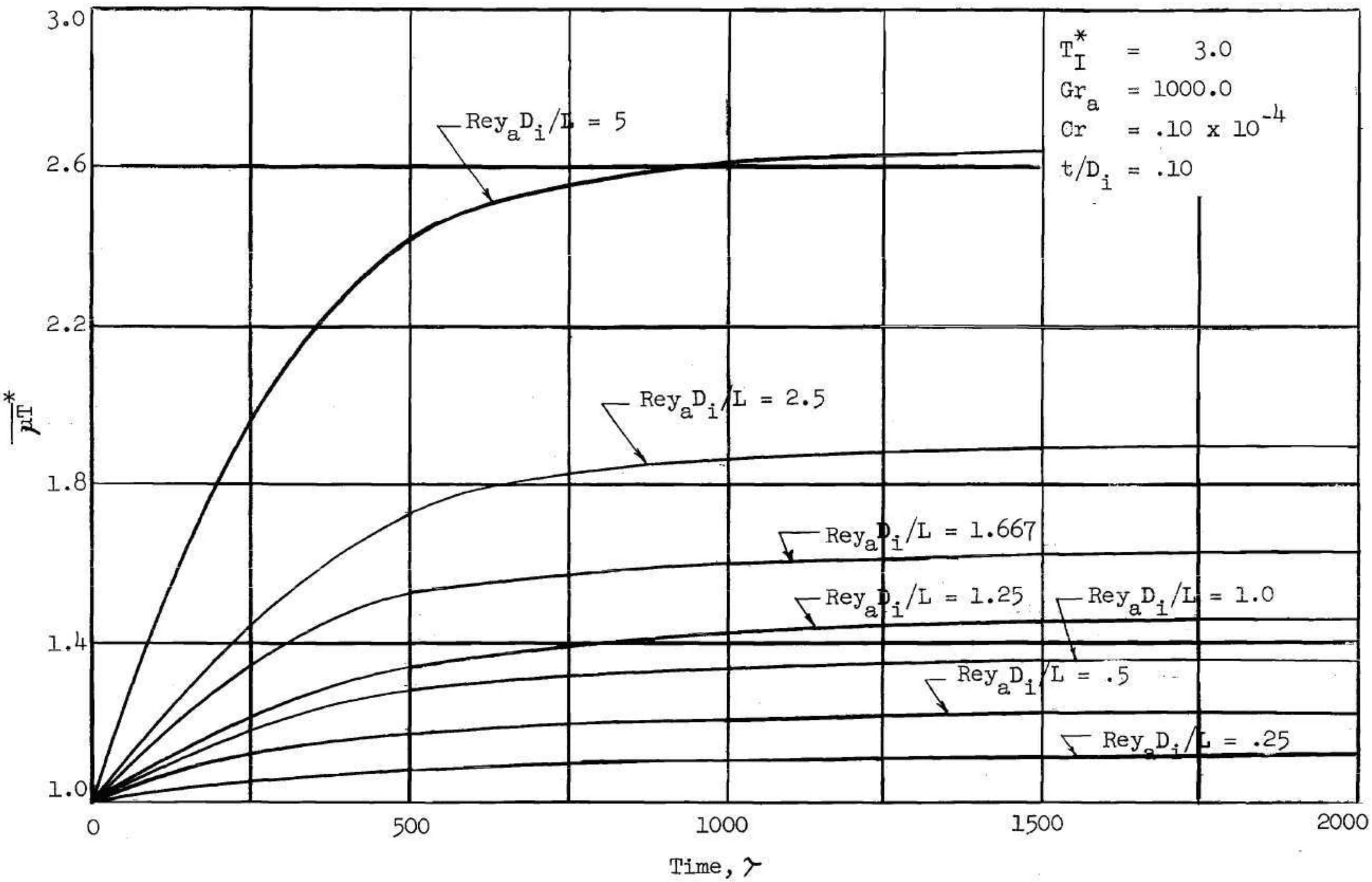

Fig. 11 The Effect of $Rey_a D_i/L$ on Variation of $\overline{\mu T}^*$ with Time, τ

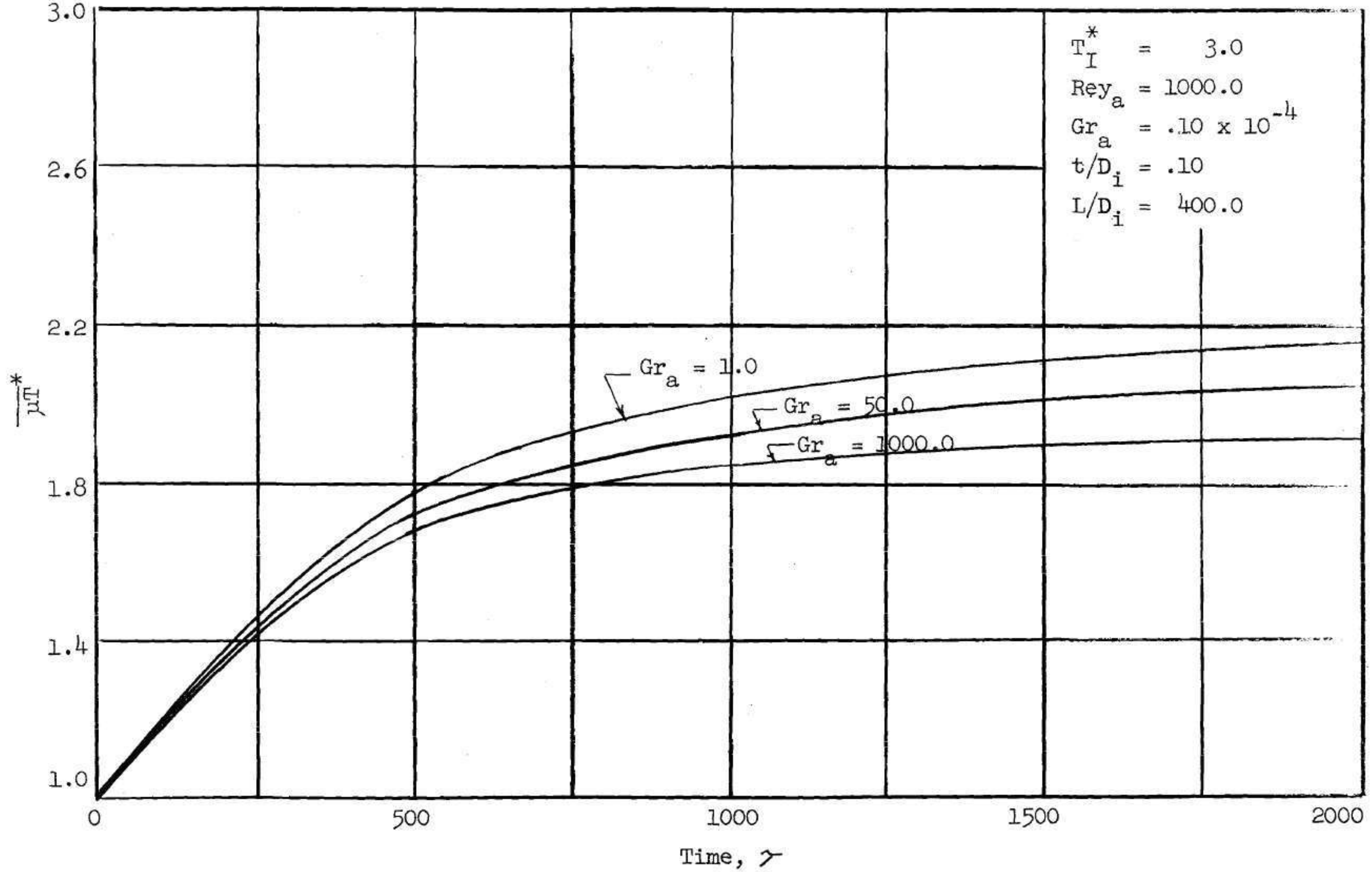

Fig. 12 The Effect of Gr_a on Variation of $\overline{\mu T}^*$ with Time, τ

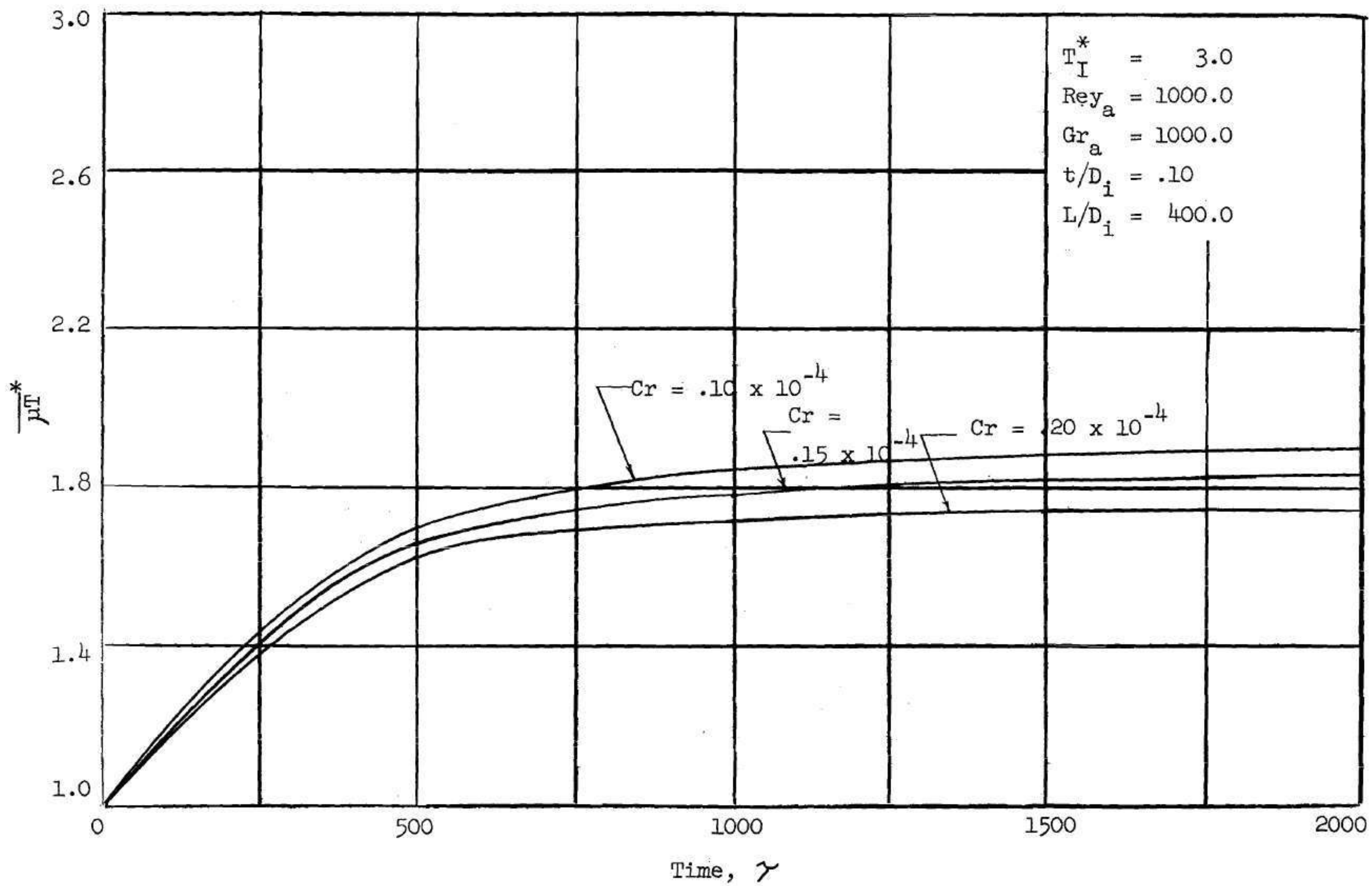

Fig. 13 The Effect of Cr on Variation of $\overline{\mu T}^*$ with Time, τ

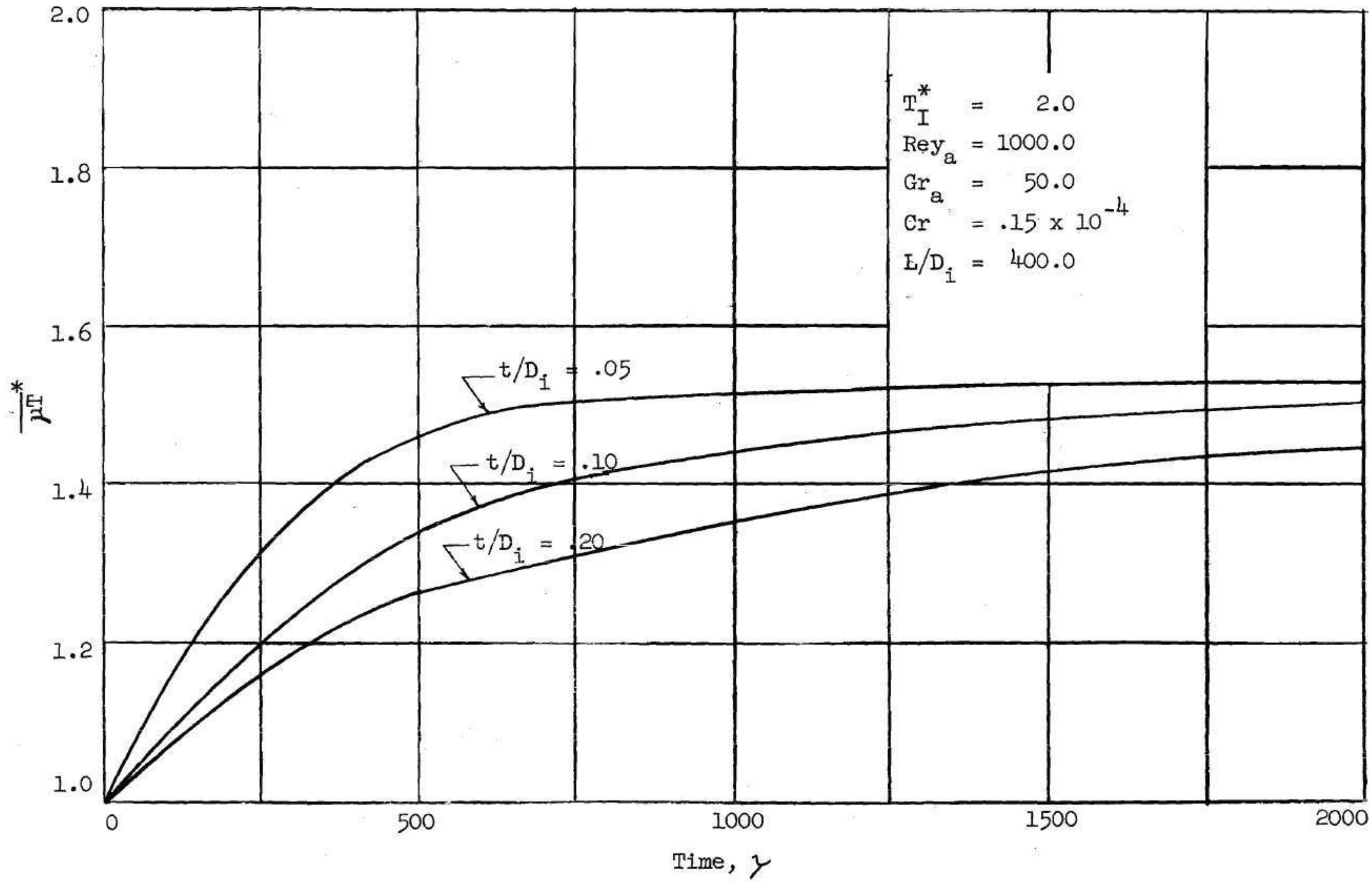

Fig. 14 The Effect of t/D_i on Variation of $\overline{\mu T}^*$ with Time, γ

variation of $\overline{\mu T}^*$ with γ since it decreases the fluid temperature distribution as shown above. These effects are seen in Figures 12, 13, and 14.

The effect of T_I^* has not been discussed here since it is obvious that as T_I^* decreases the fluid temperature and $\overline{\mu T}^*$ both decrease.

The next phase of this study will be to investigate the possibility of developing a correlation equation for $\overline{\mu T}^*$ as a function of the parameters given in Table 1.

C - Development of a Correlation Equation.--Since it is known that $\overline{\mu T}^*$ must reduce to unity when the air in the tube is constant at T_a, it was decided to investigate the existence of a correlation equation of the form

$$\overline{\mu T}^* = 1 + Z \tag{III-1}$$

where Z varies with the parameters given in Table 1 and γ. It was observed in a preliminary investigation (see Appendix D) of flow in constant wall temperature tubes with elevated inlet temperatures that $\overline{\mu T}^*$ correlated well as essentially a linear function of $ReyD_i/L$ if T_I^* approached unity for some x less than L. A plot of $\overline{\mu T}^*$ versus $ReyD_i/L$ for various combinations of the parameters in Table 1 indicated that this would be a good starting point for this investigation. It was observed that the plot was approximately a straight line so that the general equation could be written as (see Fig. 15)

$$\overline{\mu T}^* = 1 + Z_1 \, Rey_a D_i/L \tag{III-2}$$

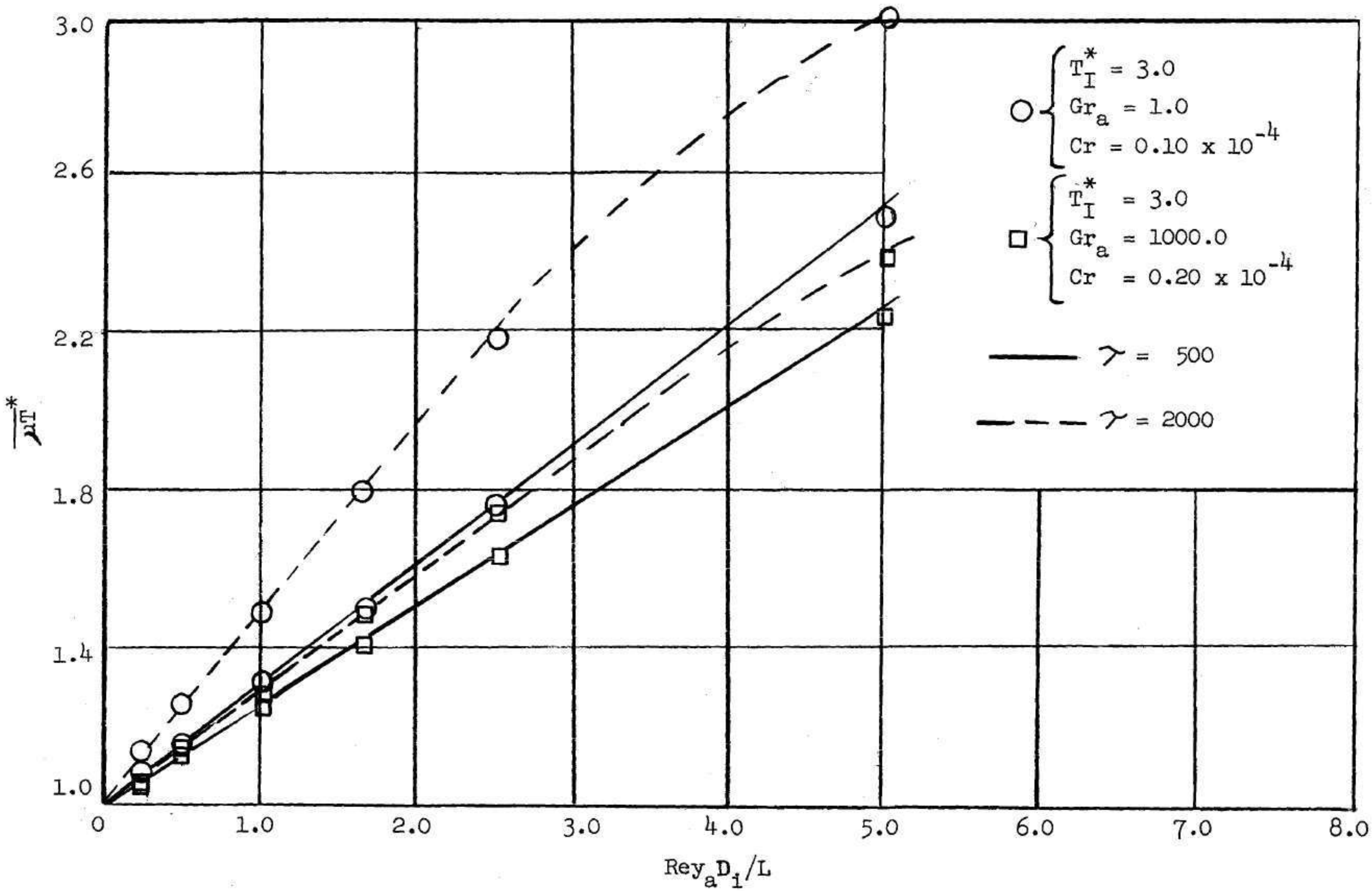

Fig. 15 Plot of $\overline{\mu T}^*$ versus $Rey_a D_i/L$

where now Z_1 varies with T_I^* , Gr_a , Cr , t/D_i , and γ .

The following procedure was used in attempting to correlate the remaining parameters. A plot of Z_1 versus a particular parameter was made for various combinations of the other parameters. For the plots of $(T_I^* - 1.0)$, Gr_a , and Cr , it was observed that they were approximately straight lines on log-log paper so that a simple power law could be written for the variations. Using an average value of the exponent for the various combinations of parameters, the following relations were obtained

$$Z_1 \propto (T_I^* - 1)^{.88}$$

$$Z_1 \propto \left(\frac{Gr_a}{50}\right)^{-.025}$$

$$Z_1 \propto \left(\frac{Cr}{.15 \times 10^{-4}}\right)^{-.18}$$

Investigation of the parameters γ and t/D_i indicated that they could not be separated as could the other parameters. Further attempts to correlate γ and t/D_i gave the following relation

$$Z_1 \propto (1 - e^{-.0017\gamma^{.7} D_i/t})$$

Combining these correlating relations into Equation (III-2) yields the following correlation equation

$$\overline{\mu T}^* = 1 + .20(1 - e^{-.0017\,\nu^{.7} D_i/t})(T_I^* - 1)^{.88} \quad \text{times}$$

$$\left(\frac{Gr_a}{50}\right)^{-.025}\left(\frac{Cr}{.15 \times 10^{-4}}\right)^{-.18} Rey_a D_i/L = 1 + Z \qquad \text{(III-4)}$$

where .20 is the constant of proportionality.

This correlation equation is accurate to within 10 per cent for all combinations of the parameters given in Table 1 except when $T_I^* = 6.0$. As was stated earlier, the assumption of a linear variation of $\overline{\mu T}^*$ with $Rey_a D_i/L$ was contingent upon T_f approaching T_a for some x less than L . For $T_I^* = 6.0$ this does not happen for L/D_i less than 400. Due to this effect the correlation equation is not as accurate for tubes of L/D_i less than 400 with $T_I^* = 6.0$.

Equation (III-4) is plotted in Figure 16. The data points shown are the values of $\overline{\mu T}^*$ obtained from the computer solution. The rectangular points correspond to values obtained for a tube of $L/D_i = 200$ with $T_I^* = 6.0$. These points are in error by as much as 20 per cent. The triangles correspond to a tube of $L/D_i = 400$ with the same inlet temperature. For this case the maximum error is reduced to 15 per cent. The circles, which correspond to all other combinations of the parameters given in Table 1, including $T_I^* = 6.0$ for L/D_i equal to 600 or greater, are within 10 per cent of the correlation equation.

It might be noted here that for application to missile sensing device systems with a possible $T_I^* = 6.0$, the connecting tubing should be of sufficient length such that T_f will approach T_a at the exit in order to protect the sensing device from damage due to high temperatures.

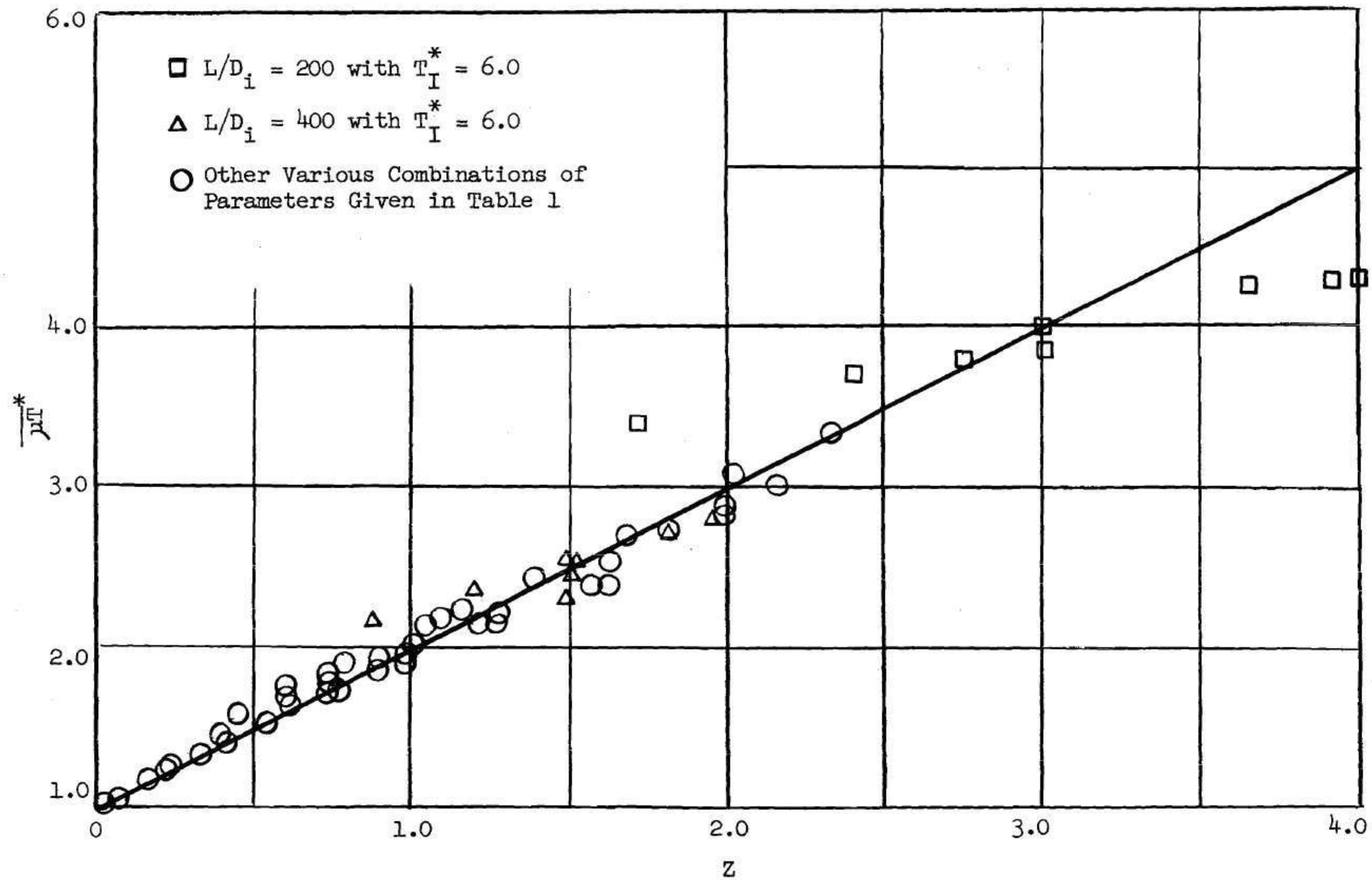

Fig. 16 Comparison of Equation (III-4) with Computer Solution

Thus, the particular case of $T_I^* = 6.0$ for $L/D_i = 200$ is not actually realistic for missile sensing device systems.

CHAPTER IV

CONCLUSIONS

The analysis obtained in this study is valid for the range of parameters given below. The system geometries analyzed consisted of circular tubing with L/D_i ratios ranging from 200 to 1000 and t/D_i ratios ranging from 0.05 to 0.20. Inlet temperature ratios may vary between 1.0 and 6.0. The flow parameters, Rey_a , Gr_a and Cr , may vary from 100 to 1000, 1 to 1000, and 0.10×10^{-4} to 0.20×10^{-4}, respectively. The non-dimensional time, γ , may be varied from 0 to 2000. The grouping of parameters, $Rey_a D_i/L$, appeared in the development of the correlation equation. This analysis is valid for variation of this parameter from 0 to 5.

Since the criterion for convergence of the finite difference equations are so important in the development of this thesis, they will be repeated here. The criteria for the finite interval $\Delta x/D_i$ was found to be related to the flow parameters through the relation,

$$\Delta x/D_i \leq \frac{Rey_a \ Pr}{2 \ Nu_i} \qquad (II\text{-}27)$$

Similarly, the criteria for the finite time interval was found to be,

$$\Delta \gamma \leq \frac{N}{Ci + Co + Cr \ T_I^{*3}} \qquad (II\text{-}42)$$

This relation places an upper limit of the value of $\Delta \gamma$. However, it shown that a lower limit on $\Delta\gamma$ had to be introduced, subject to the following criteria,

$$\frac{U\,\Delta t}{\Delta x} = 2\ Nu_i\,\Delta\gamma\ \frac{\alpha_f}{\alpha_w} \gg 1 \tag{II-13}$$

Thus, it may be concluded that as long as finite intervals are consistent with these criterion, then the finite difference equations developed will yield satisfactory results.

Since it was shown that the pressure drop across a tube is directly related to the mean product of viscosity and temperature, the various factors effecting $\overline{\mu T}^*$ were investigated. Resulting from this investigation the following correlation equation was obtained,

$$\overline{\mu T}^* = 1 + Z \tag{III-4}$$

where

$$Z = .20(1 - e^{-.0017\,\gamma^{.7} D_i/t})(T_I^* - 1)^{.88}\left(\frac{Gr_a}{50}\right)^{-.025}\left(\frac{Cr}{.15 \times 10^{-4}}\right)^{-.18} Rey_a D_i/L$$

Thus, the pressure drop equation becomes

$$\frac{p_I^2 - p_E^2}{p_a^2} = B\,\frac{L}{D_i}\,Rey_a\,(1 + Z) \tag{II-6}$$

The accuracy of Equation (III-4) was seen to be within 10 per cent of the values of $\overline{\mu T}^*$ obtained from the computer solution except for the case of $T_I^* = 6.0$ with relatively low values of L/D_i . This is attributed to the fact that T_f did not approach T_a at the exit of the tube for this particular case.

It should be pointed out that no information was obtained for T_I^* between 3.0 and 6.0. Thus, to always be assured of at least 10 per cent accuracy, the restriction placed on T_I^* , i.e., $L/D_i > 400$, should also be applied to $T_I^* = 4.0$ and 5.0. This restriction could be totally unnecessary for $T_I^* = 4.0$, but due to the lack of data no definite conclusions can be obtained concerning it.

Finally, in summary, it can be concluded from the data obtained that the correlation equation obtained in this study is a valid approximation, within the accuracy given, for the range of parameters investigated as long as the temperature of the fluid at the exit is close to the ambient temperature.

CHAPTER V

RECOMMENDATIONS

The following recommendations are made concerning this analysis:

1. An experimental investigation should be conducted to verify the theoretical analysis presented in this thesis.

2. An investigation should be made with varying inlet temperature. This investigation could be utilized to simulate a missile which is re-entering the earth's atmosphere. An analysis of this type could also investigate the accuracy of the correlation formula given in Equation (III-10) where now T_I^* would vary with time.

3. An investigation should be conducted in which a closed sensing volume is attached to the exit of the tube. This analysis, combined with the varying inlet temperature, could be utilized in analyzing the pressure lag inherent in a missile sensing device when re-entering the earth's atmosphere.

APPENDIX A

DEVELOPMENT OF THE PRESSURE LOSS EQUATION

The equation relating the pressure drop across the tube to $\overline{\mu T}$ is given by Equation (II-5)

$$p_I^2 - p_E^2 = 64 \frac{GR}{D_i} \frac{L}{D_i} \overline{\mu T} =$$

$$\frac{256\ RL}{\pi D_i^4} \overline{\mu T}\ w \tag{A-1}$$

Now define the following

$$Rey_a = \frac{4\ w\ D_i}{\pi D_i^2\ u_a} \tag{A-2}$$

where the subscript, a , denotes properties based on the temperature of the surrounding air a large distance from the tube. Using this, Equation (A-1) becomes

$$p_I^2 - p_E^2 = \left(\frac{64\ RL}{D_i^3} Rey_a\right) \overline{\mu T} \tag{A-3}$$

Multiplying and dividing Equation (A-3) by $\mu_a T_a$ and p_a^2 it becomes

$$\frac{p_I^2 - p_E^2}{p_a^2} = \frac{64\ RL}{D_i^3\ p_a^2}\ \mu_a{}^2 T_a\ Rey_a\ \overline{\mu T}^* \tag{A-4}$$

where $\overline{\mu T}^* = \overline{\mu T} / \mu_a T_a$ and other quantities are as defined earlier.

Now for the final form re-write Equation (A-4) as

$$\frac{p_I^2 - p_E^2}{p_a^2} = B \frac{L}{D_i} \mathrm{Rey}_a \, \overline{\mu T}^* \qquad \text{(A-5)}$$

where

$$B = \frac{64 \, R \, \mu_a^2 T_a}{D_i^2 \, p_a^2}$$

and is independent of the flow in the tube.

APPENDIX B

DEVELOPMENT OF EQUATION (II-24)

Considering Equation (II-20)

$$T^*_{f_{j+1}} = \frac{4\ Nu_i}{2Nu_i + \frac{Rey\ Pr}{\Delta x/D_i}} T^*_{w_j} + \frac{\frac{Rey\ Pr}{\Delta x/D_i} - 2Nu_i}{2Nu_i + \frac{Rey\ Pr}{\Delta x/D_i}} T^*_{f_j} \tag{B-1}$$

it is necessary to obtain expressions for Nu_i and $\frac{Rey\ Pr}{\Delta x/D_i}$. For the Nusselt Number Nu_i in Equation (B-1) the correlation given in Reference 6 will be used

$$Nu_i = 4.36 + \frac{0.036}{\frac{x/D_i}{Rey\ Pr} + 0.0011} \tag{B-2}$$

The constant term in Equation (B-2) is the Nusselt Number for fully developed flow. The second term accounts for development losses associated with the Langhaar (Ref. 8) velocity distribution in the inlet region of the tube. The Reynolds number, Rey , is based on the tube diameter and varies with axial position since the temperature is varying. Now define the following Reynolds number which is based on conditions of the surrounding air,

$$Rey_a = \frac{w\ D_i}{A\ \mu_a} \tag{B-3}$$

so that

$$\frac{x/D_i}{Rey\ Pr} = \left(\frac{\mu}{\mu_a}\right)\frac{x/D_i}{Rey_a\ Pr} \tag{B-4}$$

Now consider the variation of viscosity with temperature. Data taken from Reference 7 gives the following variation which is shown in Figure 3.

$$\frac{\mu_f}{\mu_a} = \left(\frac{T_f}{T_a}\right)^{0.675} \tag{B-5}$$

But, following the assumption of linearly varying temperatures

$$T_f = 1/2(T_{f_j} + T_{f_{j+1}})$$

so that

$$\frac{x/D_i}{Rey\ Pr} = \left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675} \frac{x/D_i}{Rey_a\ Pr} \tag{B-6}$$

where $T^* = T/T_a$

Also since a finite difference analysis is being used the tube is subdivided into n elements of length Δx, so that

$$x = (j - 1/2)\Delta x \tag{B-7}$$

where j number of element being considered and x is measured from the tube inlet to the midpoint of the element. Using Equations (B-6) and (B-7), Equation (B-2) takes the form

$$Nu_i = 4.36 + \frac{0.036}{(j - 1/2)\left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675} \dfrac{\Delta x/D_i}{Rey_a\ Pr}} \tag{B-8}$$

Now, using Equations (B-6) and (B-8) in Equation (B-1), the following equation is obtained,

$$T^*_{f_{j+1}} = \frac{4\ Nu_i \dfrac{\Delta x/D_i}{Rey_a\ Pr} \left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675}}{1 + 2\ Nu_i \dfrac{\Delta x/D_i}{Rey_a\ Pr} \left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675}}\ T^*_{w_j}$$

$$+ \frac{1 - 2\ Nu_i \dfrac{\Delta x/D_i}{Rey_a\ Pr} \left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675}}{1 + 2\ Nu_i \dfrac{\Delta x/D_i}{Rey_a\ Pr} \left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{0.675}}\ T^*_{f_j} \qquad \text{(B-9)}$$

APPENDIX C

DEVELOPMENT OF EQUATION (II-40)

Now consider Equation (II-32) and develop the coefficients of the various temperatures.

$$T'_{w_j} = T_{w_j} + \left(\frac{T_{f_j} + T_{f_{j+1}}}{2} - T_{w_j}\right) \frac{4\, h_i\, D_i}{\rho_w\, c_{p_w}\, (D_o^2 - D_i^2)}\, \Delta t$$

$$- (T_{w_j} - T_a) \frac{4\, h_o\, D_o}{\rho_w\, c_{p_w} (D_o^2 - D_i^2)}\, \Delta t - (T_{w_j}^{\;4} - T_a^{\;4}) \frac{4\, \varepsilon\, \sigma\, D_o}{\rho_w\, c_{p_w} (D_o^2 - D_i^2)} \qquad \text{(C-1)}$$

Investigating the term,

$$\frac{4\, h_i\, D_i}{\rho_w\, c_{p_w} (D_o^2 - D_i^2)}\, \Delta t \;,$$

and using the definition of the Nusselt Number, Nu_i , the term may be written as

$$\frac{4\, h_i\, D_i}{\rho_w\, c_{p_w} (D_o^2 - D_i^2)}\, \Delta t = \frac{4\, Nu_i\, k_f}{\rho_w\, c_{p_w}\, 2t(D_o - D_i + 2D_i)}\, \Delta t$$

$$= \frac{Nu_i\, k_f}{\rho_w\, c_{p_w} (t/D_i + t^2/D_i^{\,2}) D_i^{\,2}}\, \Delta t \qquad \text{(C-2)}$$

Now introduce, $\alpha_w = \dfrac{k_w}{\rho_w c_{p_w}}$, the thermal diffusivity of the wall into Equation (B-2)

$$\frac{Nu_i \; k_f/k_w}{(t/D_i + t^2/D_i^{\,2})} \Delta\tau \;\triangleq\; C_i \Delta\tau/N \tag{C-3}$$

where $\Delta\tau$ = non-dimensional time increment = $\dfrac{\alpha_w}{D_i^{\,2}}\Delta$

Data obtained from Reference 7 indicates the following variation of k for air with temperature as seen in Figure 4

$$k_f/k_w = \left[1/2(T^*_{f_j} + T^*_{f_{j+1}})\right]^{.875} k_a/k_w$$

Having now reduced the first term to a function of tube geometry and temperature, investigation of the second term should yield similar results. The second term may be written, using the definition of Nusselt Number, Nu_o , and following the same procedure as used in obtaining Equation (C-3), as

$$\frac{4\, h_o\, D_o\, \Delta t}{\rho_w\, c_{p_w}(D_o^{\,2} - D_i^{\,2})} = \frac{4\, Nu_o\; k_o/k_w}{(t/D_i + t^2/D_i^{\,2})}\Delta\tau \;\triangleq\; C_o \Delta\tau/N \tag{C-4}$$

where

$$k_o/k_w = \left[1/2(T^*_{w_j} + 1)\right]^{0.875} k_a/k_w$$

and

$$Nu_o = 0.95(Gr_o \cdot Pr)^{0.2}$$

in which $Gr_o = (g\beta/\nu^2)_o(\pi/2\ D_o)^3(T_{w_j} - T_a)$

Now let Gr_a = Grashoff Number based on temperature of conditions at a and the temperature difference $T_I - T_a$.

$$= (g\beta/\nu^2)_a(\pi/2\ D_o)^3(T_I - T_a)$$

Data from Reference 7 indicates the following variation of $g\beta/\nu^2$ with temperature

$$\frac{(g\beta/\nu^2)_o}{(g\beta/\nu^2)_a} = \left[1/2(T^*_{w_j} + 1)\right]^{-4.45}$$

so that

$$Gr_o = Gr_a \left[(1/2\ T^*_{w_j} + 1)\right]^{-4.45} \Delta T^* \qquad \text{(C-5)}$$

where

$$\Delta T^* = \frac{T_{w_j} - T_a}{T_I - T_a}$$

Thus, the second term is now a function of tube geometry and temperature.

The last term, radiation effect, is unwieldy due to the temperature being raised to the fourth power. However, it does simplify somewhat by factoring $T^4_{w_j} - T^4_a$.

The complete last term may be written as

$$\frac{4\varepsilon\sigma D_o}{\rho_w c_{p_w}(D_o^2 - D_i^2)} \Delta t(T_{w_j}^{\ 4} - T_a^{\ 4}) = \frac{4\varepsilon\sigma D_o}{(t/D_i + t^2/D_i^2)k_w}(T_{w_j}^{\ 3} + T_{w_j}^{\ 2} T_a$$

$$+ T_{w_j} T_a^{\ 2} + T_a^{\ 3})(T_{w_j} - T_a)\Delta\tau \qquad (C\text{-}6)$$

by following the same procedure as earlier. Multiplying and dividing by $T_a^{\ 3}$ yields the following

$$\frac{4\varepsilon\sigma D_o T_a^{\ 3}}{k_w(t/D_i + t^2/D_i^2)}(T_{w_j}^{*\ 3} + T_{w_j}^{*\ 2} + T_{w_j}^{*} + 1)(T_{w_j} - T_a)\Delta\tau \triangleq$$

$$\frac{Cr}{N}(T_{w_j}^{*\ 3} + T_{w_j}^{*\ 2} + T_{w_j} + 1)(T_{w_j} - T_a) \qquad (C\text{-}7)$$

Now, using Equations (C-3), (C-6), and (C-7) and dividing through by T_a to non-dimensionalize the temperature, Equation (C-1) becomes

$$T'^{*}_{w_j} = T^{*}_{w_j} + \frac{Ci}{N}\left(\frac{T^{*}_{f_j} + T^{*}_{f_{j+1}}}{2} - T^{*}_{w_j}\right)\Delta\tau - \frac{Co}{N}(T^{*}_{w_j} - 1)\Delta\tau - \frac{Cr}{N}(T^{*\,4}_{w_j} - 1)\Delta\tau$$

$$(C\text{-}8)$$

Thus, the temperature of the wall after a time interval can be calculated by knowing the temperatures before the time interval.

APPENDIX D

PRELIMINARY INVESTIGATION

An unpublished preliminary investigation, conducted by Dr. Frank M. White of the Aeronautical Engineering Department, for laminar flow in tubes with high inlet temperatures indicated that $\overline{\mu T}^*$ could be correlated as essentially a linear function of $Rey_a D_i/L$ if T_f^* approached unity for some x less than L. The investigation was concerned with the steady state solution of flow in constant wall temperature tubes. Also, the Reynolds analogy between heat transfer and the friction factor was used.

This discussion will outline the procedure used by Dr. White in terms of the relations developed in the body of this thesis. The simplified results obtained give information as to the type of correlation expected for the more exact analysis.

First, write the approximate relation for the viscosity effect as a simple power law temperature variation. This relation has been given before as,

$$\frac{\mu}{\mu_a} = \left(\frac{T}{T_a}\right)^{0.675}$$

This approximation may be used in the calculation of $\overline{\mu T}^*$:

$$\overline{\mu T}^* = \frac{\overline{\mu T}}{\mu_a T_a} = 1/L \int_0^L \frac{\mu T}{\mu_a T_a}\, dx = \int_0^1 \left(\frac{T}{T_a}\right)^{1.675} d(x/L) \qquad \text{(D-1)}$$

In this investigation, the subscript a refers to the ambient conditions surrounding the tube, as stated in Chapter II.

From Equation (D-1), it is seen that the problem of determining $\overline{\mu T}^*$ is that of determining the temperature distribution in the tube at any given instant. The simplified theory which follows attempts to do this.

First, write Equation (II-15) in its differential form,

$$h_i \pi D_i \, dx(T_f - T_w) = - w \, c_{p_f} \, d \, T_f \tag{D-2}$$

where the minus sign is introduced since dT_f is negative and the heat transferred to the element is positive. Now introduce the following correlation numbers:

$Nu_i = h_i D_i / k_f$

$Rey = w D_i / A \mu$

and $Pr = \mu \, c_p / k$

Thus Equation (D-2) becomes

$$dT_f = \frac{4 \, Nu_i}{Rey \, Pr} (T_w - T_f) \frac{dx}{D_i} \tag{D-3}$$

Now assume that the wall temperature is approximately equal to the ambient temperature, $T_w \simeq T_a$, constant along the tube. Using an approximate average value of $Nu_i \simeq 5.0$, which comes from consideration of Equation (II-23), and $Pr = 0.72$, Equation (D-3) becomes

$$d \, T_f/T_a \cong \frac{28}{Rey} (1 - T_f/T_a) \frac{dx}{D_i} \tag{D-4}$$

which may be integrated readily if the Reynolds number is assumed to be approximately constant:

$$\int_{T_I/T_a}^{T_f/T_a} \frac{d\ T_f/T_a}{(1 - T_f/T_a)} \cong \frac{28}{\text{Rey}\ D_i} \int_o^x dx \tag{D-5}$$

Integrating the above equation and re-arranging gives the following expression for T_f at any point x :

$$T_f/T_a \cong 1 + \left[(T_I/T_a) - 1\right] e^{-\frac{28\ L/D_i}{\text{Rey}} \left(\frac{x}{L}\right)} \tag{D-6}$$

This relation is a simplified approximation to the theoretical temperature distribution in the tube.

Equation (D-6) may be used to calculate $\overline{\mu T}^*$:

$$\overline{\mu T}^* = \int_o^1 \left\{ 1 + \left[T_I/T_a - 1\right] e^{-\frac{28\ L/D_i}{\text{Rey}} \left(\frac{x}{L}\right)} \right\}^{1.675} d\left(\frac{x}{L}\right) \tag{D-7}$$

Again, it is suggested that Rey be assumed constant to simplify the integration. An explicitly closed form for this integral has not yet been found. However, for a given value of T_I/T_a , L/D_i , and Rey one can easily evaluate the integral numerically. Performing this and making a plot of $\overline{\mu T}^*$ versus Rey D_i/L for a constant value of T_I/T_a results in an approximate linear variation if the fluid temperature T_f approaches T_a as x approaches L . An example of a plot of this type is given in Figure 17 for $T_I/T_a = 5.0$.

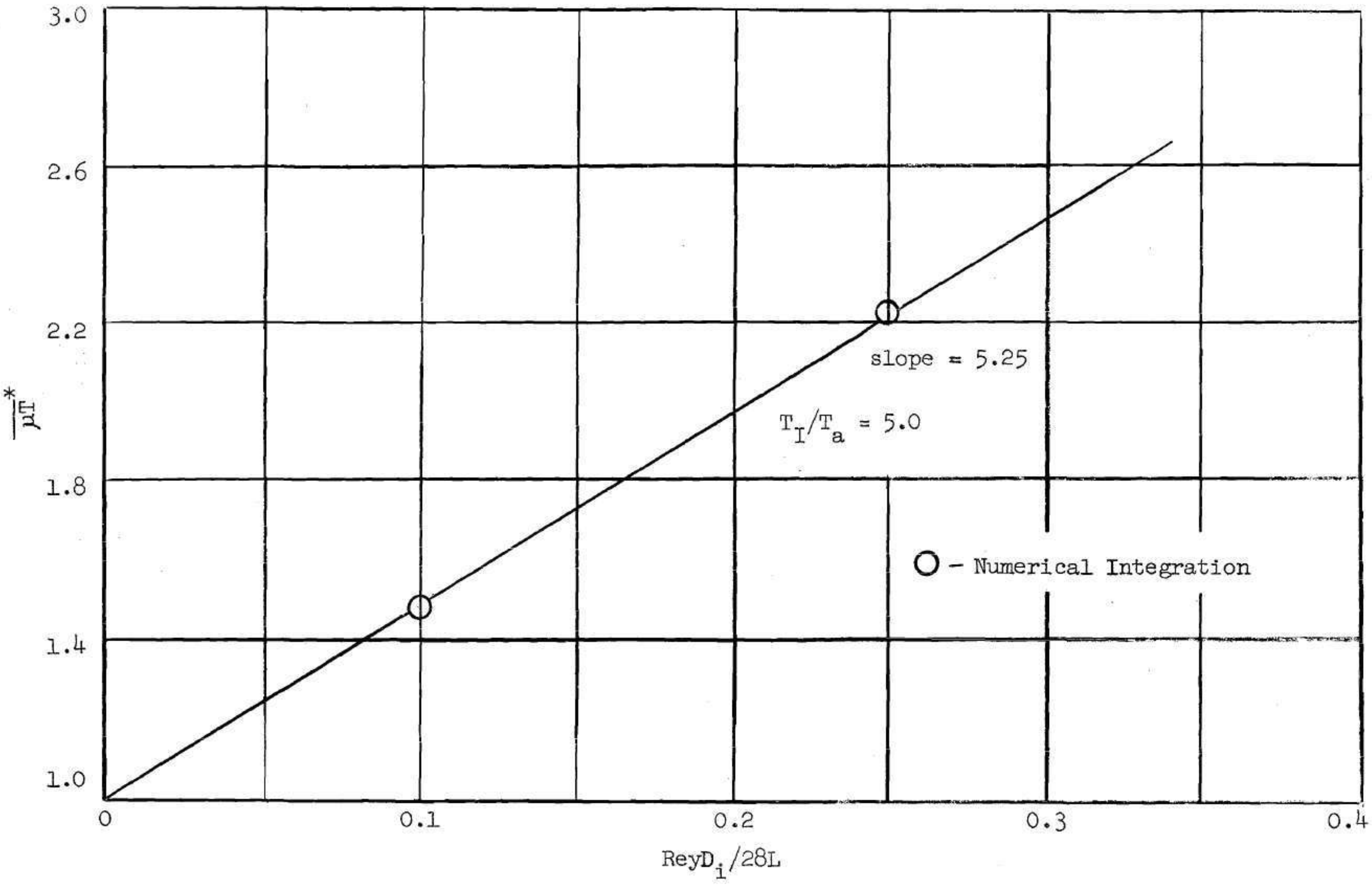

Fig. 17 Variation of $\overline{\mu T}^*$ with $ReyD_i/28L$

Thus the general equation may be written,

$$\overline{\mu T}^{*} = 1 + M \text{ Rey } D_i/L \tag{D-8}$$

where M varies with T_I/T_a . Dr. White's investigation indicated also that M could be correlated by a power law of $(T_I/T_a - 1)$.

These two results were then chosen as starting points for the development of a correlation equation for the more exact study presented in this thesis.

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